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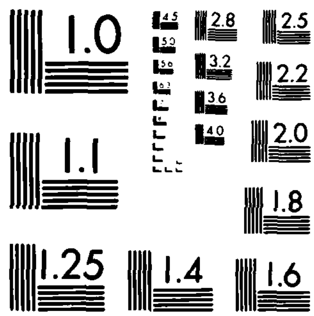

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Report SD-TR-86-29

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UNIFORM VERSUS GAUSSIAN BEAMS: A COMPARISON OF THE EFFECTS OF DIFFRACTION, OBSCURATION, AND ABERRATIONS

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16 December 1985

Interim Report

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This interim report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-85-C-0086 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by J. R. Parsons, Principal Director, Sensor Systems Subdivision and J. R. Stevens, Principal Director, Advanced Programs. The Air Force project officer was Captain Michael D. Zwart.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This techical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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Diffraction	Weakly truncated Gaussian beam	Encircled energy
Circular aperture	Focused beam	Point spread function
Annular aperture	Collimated beam	Apodization
Uniform beam	Aberrations (spherical, coma, astigmatism)	

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Much is said in the literature about Gaussian beams, but little is said in terms of a quantitative comparison between the propagation of uniform and Gaussian beams. Even when results for both types of beams are given, they appear in a normalized form in such a way that some of the quantitative difference between them is lost. ~~In~~ this report, ~~we first~~ consider an aberration-free annular beam and investigate the effect of Gaussian amplitude across the aperture on the focal-plane irradiance and encircled-power distributions. The axial irradiance of both uniform and Gaussian focused beams —

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19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

is calculated, and the problem of optimum focusing is discussed. The results for a collimated beam are obtained as a limiting case of a focused beam. Next, ~~we~~ consider the problem of aberration balancing and compare the effects of primary aberrations on the two types of beams. Finally, the limiting case of weakly truncated Gaussian beams is discussed, and simple results are obtained for the irradiance distribution and the balanced aberrations.

PREFACE

The author gratefully acknowledges computational assistance by A. Compito and C. Myers, plotting by C. Sheffer, and typing by R. Wallace. Some of this work was presented at the SPIE August 1985 meeting held in San Diego, California, and will appear in the SPIE Proceedings, Vol. 560, entitled "Diffraction Phenomena in Optical Engineering Applications." The complete work reported here will also appear in a feature issue of the Journal of the Optical Society of America, April 1986, entitled "Propagation and Scattering of Beam Fields."

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FIGURES

TABLES

1. INTRODUCTION

The propagation of Gaussian beams has been extensively explored in the literature. Most of the early literature neglects the effects of beam truncation by an aperture with the result that a beam remains Gaussian as it propagates.[1-5] Often, it is the propagation of collimated beams that is discussed,[6-10] but lately the emphasis has been on focused beams.[11-13] With few exceptions,[11,14-16] the discussion is limited to aberration-free circular beams.

In spite of the vast amount of literature on the propagation of Gaussian beams, there is little in terms of a quantitative comparison with the propagation of a uniform beam. Even when results for both types of beams are occasionally given,[6-8,10,11] they appear in a normalized form in such a way that some of their quantitative difference is lost. The objective of this report is to compare quantitatively the effects of diffraction, obscuration, and aberrations on the propagation of uniform and Gaussian beams. Equations are derived for the axial and transverse irradiance distributions for a focused beam. Equations for encircled-power distributions are also given. For a meaningful comparison between a uniform and a Gaussian beam, the total power transmitted by the aperture is kept fixed, regardless of the value of the obscuration or the nature of the beam. It is shown that, in the focal plane, the irradiance at the focus and in its vicinity is smaller for a Gaussian beam

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[1] Gaskill, J. D., Linear Systems, Fourier Transforms, and Optics, John Wiley and Sons, NY, 1978, Section 10-7.
[2] Siegman, A. E., An Introduction to Lasers and Masers, McGraw-Hill Book Company, NY, 1971, Section 8-2.
[3] Dickson, L. D., "Characteristics of a Propagating Gaussian Beam," Appl. Opt., 9, 1970, pp. 1854-1861. (This paper considers the effects of aperture truncation and derives a condition under which they may be neglected.)
[4] Williams, C. S., "Gaussian Beam Formulas from Diffraction Theory," Appl. Opt., 12, 1973, pp. 871-876.
[5] Herman, R. M., J. Pardo, and T. A. Wiggins, "Diffraction and Focusing of Gaussian Beams," Appl. Opt., 24, 1985, pp. 1346-1354.
[6] Buck, A. L., "The Radiation Pattern of a Truncated Gaussian Aperture Distribution," Proc. IEEE, 55, 1967, pp. 448-450.

than that for a uniform beam. However, away from the focus but within the central disc of the diffraction pattern, the irradiance is higher for a Gaussian beam. Accordingly, the encircled power is higher for a uniform beam for very small circles, but the reverse is true for larger circles. The results for a collimated beam are obtained as a limiting case of a focused beam, namely, a beam focused at infinity. The differences between the diffraction patterns for uniform and Gaussian beams decrease as the obscuration increases.

The problem of aberration balancing is discussed for the two types of beams. Zernike polynomials representing balanced primary aberration for uniform and Gaussian annular beams are described. The relationship between the peak value of a primary aberration and its corresponding standard deviation across a circular aperture is tabulated for both uniform and Gaussian beams. It is shown that a Gaussian beam for which the irradiance at the edge of the aperture is e^{-2} times the irradiance at its center (if there were no obscuration) is only somewhat less sensitive to aberrations than a corresponding uniform beam. For a Gaussian beam with a much weaker truncation by the aperture, the aberration tolerance increases rapidly. However, in that case, since the beam power is concentrated in a small region near the center of the aperture, the effect of aberration in its outer region is negligible. Accordingly, for a weakly truncated Gaussian beam, the aberration coefficients are defined in terms of the peak aberration at the e^{-2} irradiance point in the aperture rather than at its edge.

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[7] Campbell, J. P., and L. G. DeShazer, "Near Fields of Truncated Gaussian Apertures," J. Opt. Soc. Am., 59, 1969, pp. 1427-1429.
[8] Olaofe, G. O., "Diffraction by Gaussian Apertures," J. Opt. Soc. Am., 60, 1970, pp. 1654-1657.
[9] Schell, R. G., and G. Tyra, "Irradiance from an Aperture with Truncated-Gaussian Field Distribution," J. Opt. Soc. Am., 61, 1971, pp. 31-35.
[10] Nayyar, V. P., and N. K. Verma, "Diffraction by Truncated-Gaussian Annular Apertures," J. Optics, 9, (Paris), 1978, pp. 307-310.
[11] Holmes, D. A., J. E. Korka, and P. V. Avizonis, "Parametric Study of Apertured Focused Gaussian Beams," Appl. Opt., 11, 1972, pp. 565-574.
[12] Li, Y., and E. Wolf, "Focal Shift in Focused Truncated Gaussian Beams," Opt. Comm., 42, 1982, pp. 151-156.
[13] Tanaka, K., N. Saga, and K. Hauchi," Focusing of a Gaussian Beam Through a Finite Aperture Lens," Appl. Opt., 24, 1985, pp. 1098-1101.

Note that the diffraction effects of a uniform or a Gaussian beam are equivalent to those of a uniform or Gaussian-apodized pupil, respectively, in an imaging system. Thus, for example, an irradiance distribution transverse to the direction of beam propagation represents a corresponding point-spread function of an imaging system.

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[14] Lowenthal, D. D., "Marechal Intensity Criteria Modified for Gaussian Beams," Appl. Opt., 13, 1974, pp. 2126-2133, 2774.

[15] Lowenthal, D. D., "Far-Field Diffraction Patterns for Gaussian Beams in the Presence of Small Spherical Aberrations," J. Opt. Soc. Am., 65, 1975, pp. 853-855.

[16] Sklar, E., "Effects of Small Rotationally Symmetrical Aberrations on the Irradiance Spread Function of a System with Gaussian Apodization Over the Pupil," J. Opt. Soc. Am., 65, 1975, pp. 1520-1521.

2. DIFFRACTION EQUATIONS

Consider an optical beam of wavelength λ diffracted by an aperture. If $U(\vec{\rho})$ represents the amplitude at a point $\vec{\rho}$ on the plane of the aperture, then, in the Fresnel approximation, the amplitude at a point $\vec{r}$ in an observation plane parallel to and at a distant z from the aperture plane is given by[17]

$$U(\vec{r};z) = \frac{\exp[ik(z + r^2/2z)]}{i\lambda z} \int U(\vec{\rho}) \exp(ik\rho^2/2z) \exp(-ik\vec{\rho} \cdot \vec{r}/z) d\vec{\rho} \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number of the beam radiation, $r = |\vec{r}|$, and $\rho = |\vec{\rho}|$. The irradiance distribution in a certain plane is equal to the square of the modulus of the corresponding amplitude distribution. For example, in the observation plane, it is given by

$$I(\vec{r};z) = |U(\vec{r};z)|^2 \quad (2)$$

The corresponding encircled power (or energy) $P(r_o)$, i.e., the power in a circle of radius r_o centered at $\vec{r} = 0$ in the observation plane, is given by

$$P(r_o;z) = \int_{|\vec{r}| \leq r_o} I(\vec{r};z)\, d\vec{r} \quad (3)$$

[17]Mahajan, V. N., "Axial Irradiance and Optimum Focusing of Laser Beams," Appl. Opt., 22, 1983, pp. 3042-3053.

3. APERTURE DISTRIBUTION

Consider an annular aperture of inner and outer radii of εa and a, respectively, where $0 \leq \varepsilon < 1$ is the linear obscuration of the aperture. As indicated in Figure 1, a beam of outer and inner radii a and εa, respectively, is focused at a distance R from the aperture plane. The beam is aberration-free when a spherical wavefront of the radius of curvature R is centered at the point of observation and passes through the center of the aperture. For a fixed total power P_o transmitted by the aperture regardless of the value of ε or the nature of illumination (uniform or Gaussian), the irradiance distribution at the aperture for uniform and Gaussian illumination may be written

$$I_u(\rho) = (P_o/A)/(1 - \varepsilon^2) \tag{4}$$

and

$$I_g(\rho) = (P_o/A)\ f(\gamma;\varepsilon)\ \exp(-2\gamma\rho^2) \tag{5}$$

respectively. Here

$$A = \pi a^2 \tag{6}$$

is the area of the unobscured circular aperture

$$f(\gamma;\varepsilon) = 2\gamma/(e^{-2\gamma\varepsilon^2} - e^{-2\gamma}) \tag{7}$$

and ρ is in units of a so that $\varepsilon \leq \rho \leq 1$. The parameter γ characterizes the truncation of a Gaussian beam by the aperture. If we define a Gaussian beam radius ω as the radial distance from the beam center at which the irradiance is equal to e^{-2} of the value at the center (if there were no obscuration), then

$$\gamma = (a/\omega)^2 \tag{8}$$

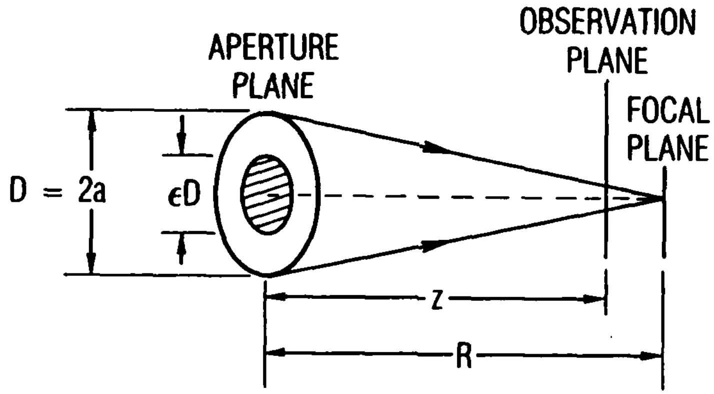


Figure 1. Diffraction Geometry

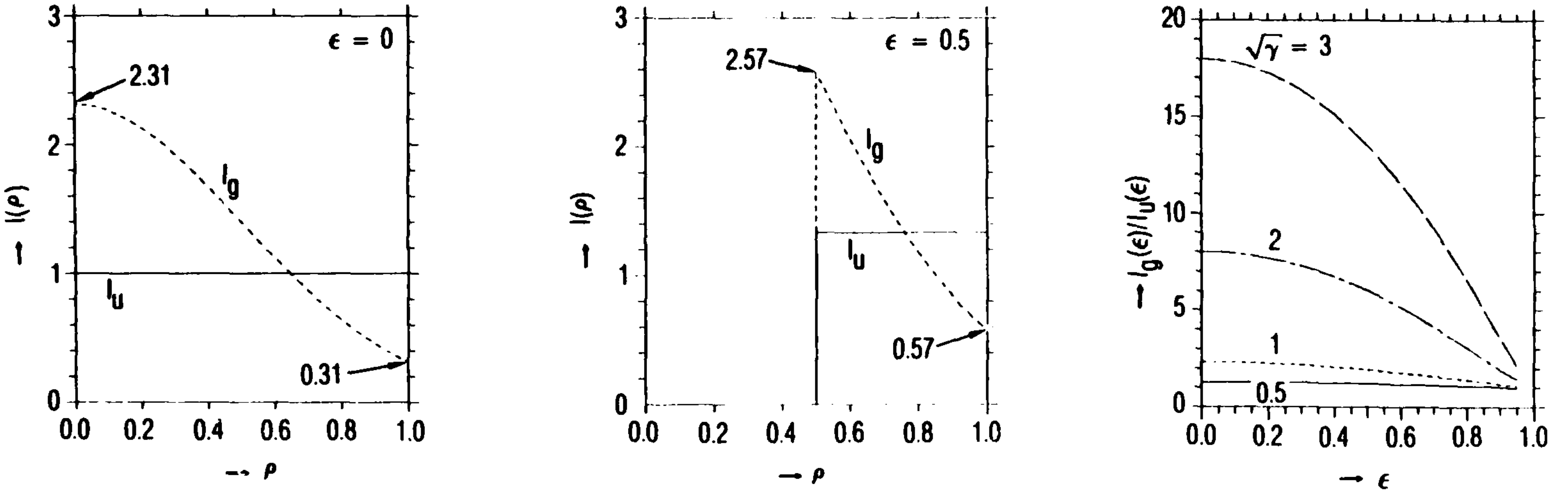
i.e., $\sqrt{\gamma}$ represents the ratio of the aperture radius and Gaussian beam radius. Note that as $\gamma \to 0$, $f(\gamma;\varepsilon) \to (1 - \varepsilon^2)^{-1}$ and Eq. (5) reduces to Eq. (4). Hence, the results for a Gaussian beam reduce to those for a corresponding uniform beam in the limit $\gamma \to 0$.

It is evident from Eq. (4) that the aperture irradiance (in units of P_o/A) for a uniform beam increases with ε as $(1 - \varepsilon^2)^{-1}$. For a Gaussian beam, the irradiance decreases exponentially from a maximum value of $f(\gamma;\varepsilon)\exp(-2\gamma\varepsilon^2)$ at the inner edge of the aperture to a minimum value of $f(\gamma;\varepsilon)\exp(-2\gamma)$ at the outer edge. For example, as illustrated in Figure 2a, when $\gamma = 1$ and $\varepsilon = 0$, the maximum and minimum values are 2.31 and 0.31, respectively. Thus, the mirrors in a high-power Gaussian beam train are not only illuminated unevenly, but they must also withstand considerably higher irradiance levels compared to those for a uniform beam. For an annular beam, the peak irradiance is even higher, as illustrated in Figure 2b for $\gamma = 1$ and $\varepsilon = 0.5$, since the same total power is now distributed across a smaller area. The ratio of the peak values of the aperture irradiance for Gaussian and uniform beams is given by

$$I_g(\varepsilon)/I_u(\varepsilon) = (1 - \varepsilon^2)\ f(\gamma;\varepsilon)\exp(-2\gamma\varepsilon^2)$$

$$= 2\gamma(1 - \varepsilon^2)/\{1 - \exp[-2\gamma(1 - \varepsilon^2)]\} \tag{9}$$

The variation of this ratio with ε is shown in Figure 2c for $\sqrt{\gamma} = 0.5$, 1, 2, and 3. It is evident that as $\gamma(1 - \varepsilon^2)$ increases, the ratio approaches a value of $2\gamma(1 - \varepsilon^2)$.



(a) Circular Aperture

(b) Annular Aperture with $\varepsilon = 0.5$

(c) Ratio $I_g(\varepsilon)/I_u(\varepsilon)$ of Peak Values of Aperture Irradiance as a Function of ε for Several Values of γ

Note: Units of aperture irradiance are P_o/A, where A is the area of the circular aperture.

Figure 2. Aperture Irradiance Distribution for Uniform and Gaussian ($\gamma = 1$) Beams of a Given Total Power P_o

4. FOCUSED BEAM

For an aberration-free beam focused on the axis of the aperture at a distance R from it, the amplitude at the aperture can be written

$$U(\rho) = \sqrt{I(\rho)}\, \exp(-iA\rho^2/\lambda R) \tag{10}$$

If, for example, R_b is the radius of curvature of a spherical wave incident on a lens of focal length f, then $R^{-1} = R_b^{-1} + f^{-1}$. Substituting Eq. (10) into Eq. (1), we find that the irradiance distribution in an observation plane can be written

$$I(r;z) = 4(R/z)^2 \left| \int_{\epsilon}^{1} \sqrt{I(\rho)}\, \exp[i\Phi_2(\rho)]\, J_0(\pi r\rho)\, \rho d\rho \right|^2 \tag{11}$$

where

$$\Phi_2(\rho) = \frac{A}{\lambda}\left(\frac{1}{z} - \frac{1}{R}\right)\rho^2 \tag{12}$$

represents the defocus phase aberration of a beam focused at a distance R with respect to a reference sphere centered at a distance z and passing through the center of the aperture. In Eq. (11), various quantities are normalized by the parameters for a uniformly illuminated circular aperture. The $I(\rho)$ is in units of P_o/A, $I(r;z)$ is in units of P_oA/λ^2R^2, ρ is in units of a, and $r = |\vec{r}|$ is in units of $\lambda z/D$, where $D = 2a$.

We shall refer to the observation plane at $z = R$ where the beam is focused at as the focal plane and the axial point at $z = R$ as the focus or the focal point. Similarly, we shall refer to a beam with $R = \infty$ at the exit of the aperture as a collimated beam. In imaging applications, the focal plane will be more appropriately called an image plane, unless the object is at infinity.

4.1 FOCAL-PLANE DISTRIBUTION

If we let $z = R$ and substitute aperture distributions, Eqs. (4) and (5), for uniform and Gaussian beams into Eq. (11), we obtain the corresponding focal-plane distributions. In the case of a uniform beam, it is given by[18,19]

$$I_u(r;R) = (1 - \varepsilon^2)^{-1} \left[\sqrt{I_c(r)} - \varepsilon^2 \sqrt{I_c(\varepsilon r)}\right]^2 \tag{13}$$

where

$$I_c(r) = [2J_1(\pi r)/\pi r]^2 \tag{14}$$

represents the focal-plane irradiance distribution for a uniform circular ($\varepsilon = 0$) beam. It is evident that for the focal-plane distribution, the variable r is in units of λF, where $F = R/D$ is the f-number of the focused beam. The encircled power, in units of P_o, is given by

$$P_u(r_o;R) = (1 - \varepsilon^2)^{-1}\left[P_c(r_o) + \varepsilon^2 P_c(\varepsilon r_o) - 4\varepsilon \int_0^1 J_1(\pi r r_o) J_1(\pi \varepsilon r r_o) dr/r\right] \tag{15}$$

where r_o is in units of λF, and

$$P_c(r_o) = 1 - J_0^2(\pi r_o) - J_1^2(\pi r_o) \tag{16}$$

represents the focal-plane encircled-power distribution in the case of a uniform circular beam.

The corresponding results for a Gaussian beam are given by

$$I_g(r;R) = 4f(\gamma;\varepsilon)\left[\int_\varepsilon^1 e^{-\gamma\rho^2} J_0(\pi r\rho)\rho d\rho\right]^2 \tag{17}$$

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[18] Born, M., and E. Wolf, Principles of Optics, Pergamon Press, NY, 1975, p. 416.

[19] Mahajan, V. N., "Included Power for Obscured Circular Pupils," Appl. Opt., 17, 1978, pp. 964-968.

and

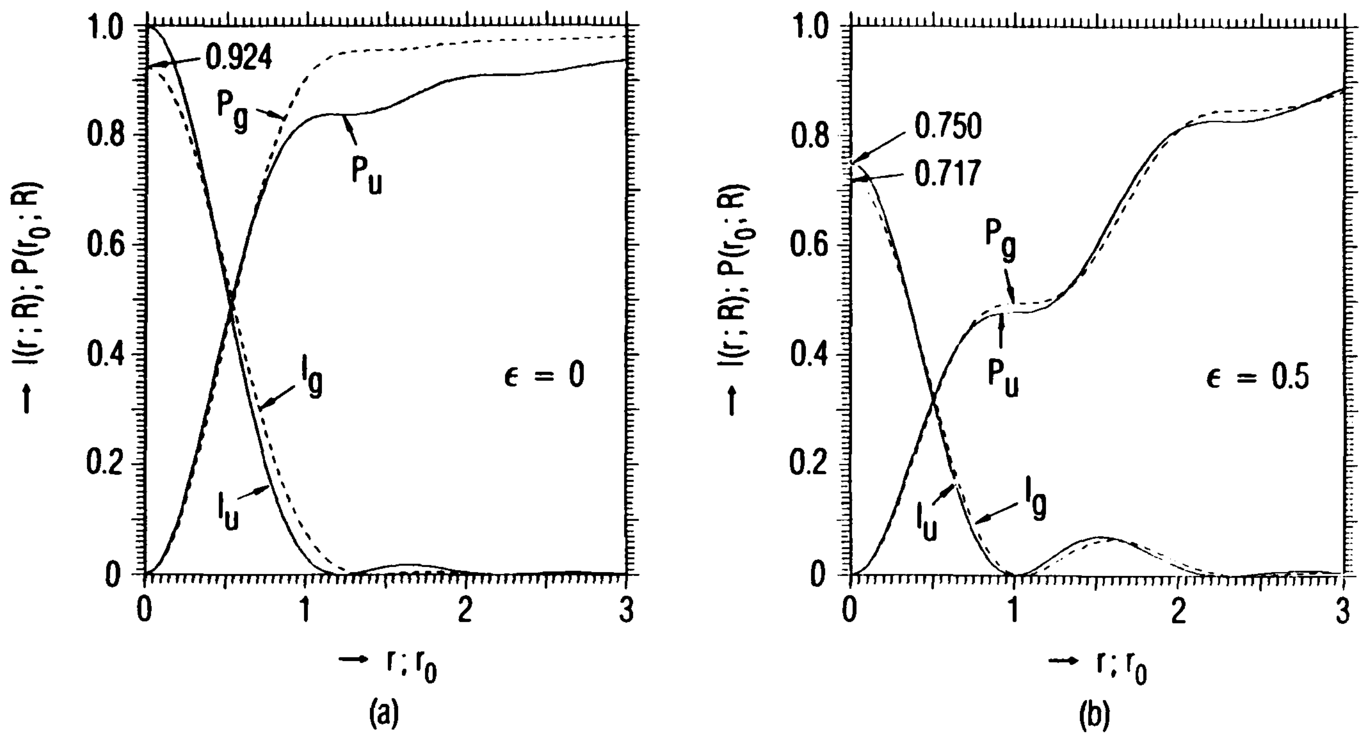
$$P_g(r_o;R) = (\pi^2/2)\int_0^{r_o} I_g(r;R)rdr \tag{18}$$

Figure 3 shows how the irradiance and encircled-power distributions in the focal plane compare for the two beams. The uniform beam corresponds to $\gamma = 0$, and the Gaussian beam considered in Figure 3 is for $\gamma = 1$. At and near the focal point, a uniform beam gives a higher irradiance than a Gaussian beam. For a circular aperture (Fig. 3a), $I_u > I_g$ for $r < 0.42$. For larger values of r, $I_g > I_u$, except in the secondary rings where again $I_u > I_g$, as is well known. The encircled power $P_u \gtrless P_g$ for $r_o \lesssim 0.63$. Of course, as $r_o \to \infty$, $P_u \to P_g \to 1$. The Gaussian illumination broadens the central disc but reduces the power in the secondary rings. For annular apertures with $\varepsilon = 0.5$ (Fig. 3b), the differences between the focal-plane distributions for uniform and Gaussian beams are less compared to those for a circular aperture. The obscuration reduces the focal-point irradiance, reduces the power in the central disc, and spreads it into the secondary rings of the diffraction pattern. It reduces the size of the central disc also. Moreover, the difference in encircled powers $P_u - P_g$ changes its sign from positive to negative to positive as r_o increases. Note also that because of the obscuration, the secondary maxima are higher and of nearly equal value for the two types of beams.

For clarity, the irradiance distributions are also plotted on a logarithmic scale (Figs. 3c and 3d). The positions of maxima and minima and the corresponding irradiance and encircled-power values are given in Table 1. It is evident that the corresponding maxima and minima for a Gaussian beam are located at higher values of r than those for a uniform beam. Moreover, whereas the principal maximum for a Gaussian beam is only slightly lower (0.924 compared to 1), its secondary maxima are lower by a factor >3 compared to the corresponding maxima for a uniform beam.

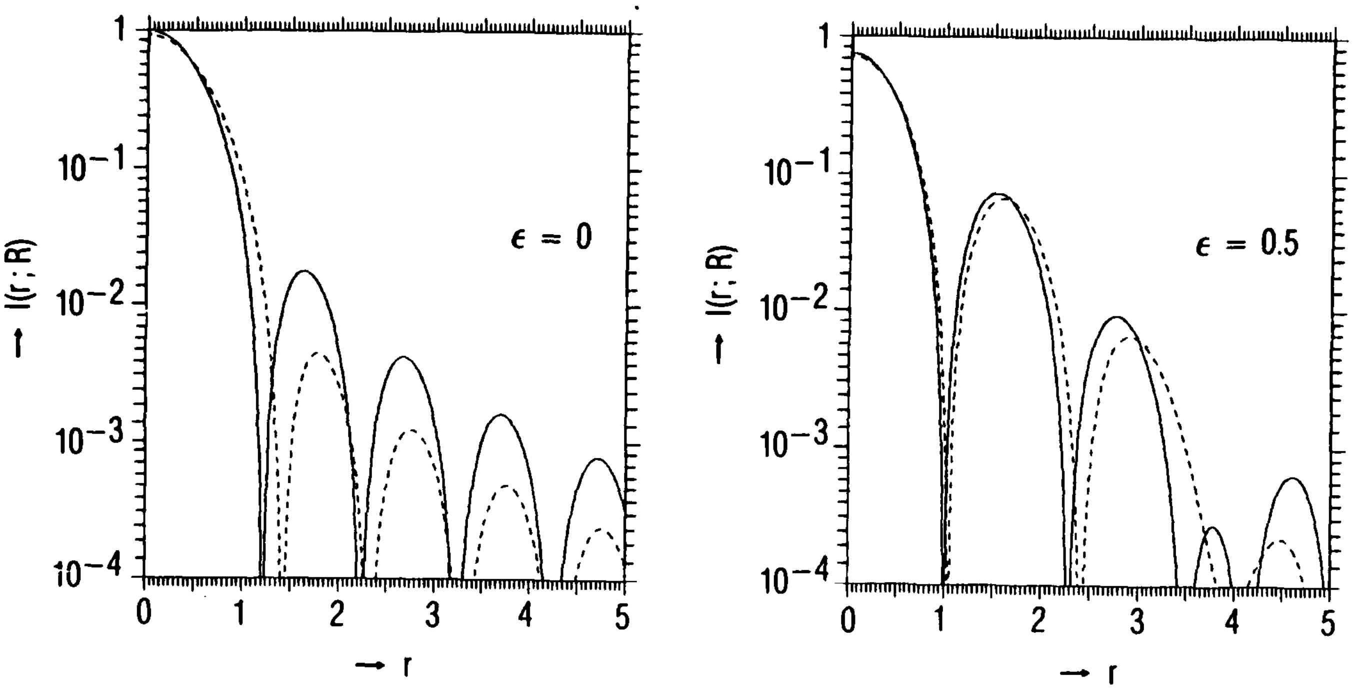
The focal-point irradiance corresponding to uniform and Gaussian beams is given by

$$I_u(0;R) = 1 - \varepsilon^2 \tag{19}$$



Note: The irradiance and encircled power are in units of P_oA/λ^2R^2 and P_o, respectively. The radial distance r or r_o in the focal plane is in units of $\lambda R/D$. The focal point is at $r = 0$.

Figure 3. Focal-Plane Irradiance and Encircled-Power Distributions for Uniform and Gaussian ($\gamma = 1$) Beams of a Given Total Power P_o



(c) (d)

Figure 3. Focal-Plane Irradiance and Encircled-Power Distributions for Uniform and Gaussian ($\gamma = 1$) Beams of a Given Total Power P_o (Continued)

Table 1. Maxima and Minima of Focal-Plane Irradiance Distribution and Corresponding Encircled Powers for Uniform and Gaussian (γ = 1) Circular Beams

Maxima/Minima	r, r_o	$I(r)$	$P(r_o)$
Max	0 (0)	1 (0.924)	0 (0)
Min	1.22 (1.43)	0 0	0.838 (0.955)
Max	1.64 (1.79)	0.0175 (0.0044)	0.867 (0.962)
Min	2.23 (2.33)	0 (0)	0.910 (0.973)
Max	2.68 (2.76)	0.0042 (0.0012)	0.922 (0.976)
Min	3.24 (3.30)	0 (0)	0.938 (0.981)
Max	3.70 (3.76)	0.0016 (0.0005)	0.944 (0.983)
Min	4.24 (4.29)	0 (0)	0.952 (0.985)
Max	4.71 (4.75)	0.0008 0.0002	0.957 (0.986)

Note: The numbers without parentheses are for a uniform beam and those with parentheses are for a Gaussian (γ = 1) beam.

and

$$I_g(0;R) = (2/\gamma) \tanh[(1 - \varepsilon^2)\gamma/2] \tag{20}$$

respectively. Their ratio is given by[20]

$$\eta = I_g(0;R)/I_u(0;R)$$

$$= \frac{\tanh[(1 - \varepsilon^2)\gamma/2]}{(1 - \varepsilon^2)\gamma/2} \tag{21}$$

Figure 4a shows how η varies with γ for several values of ε. It is evident that η decreases as γ increases, regardless of the value of ε. However, as shown in Figure 4b, for a given value of γ, η increases as ε increases. Note that for large values of γ

$$\eta \to 2/\gamma(1 - \varepsilon^2) \tag{22}$$

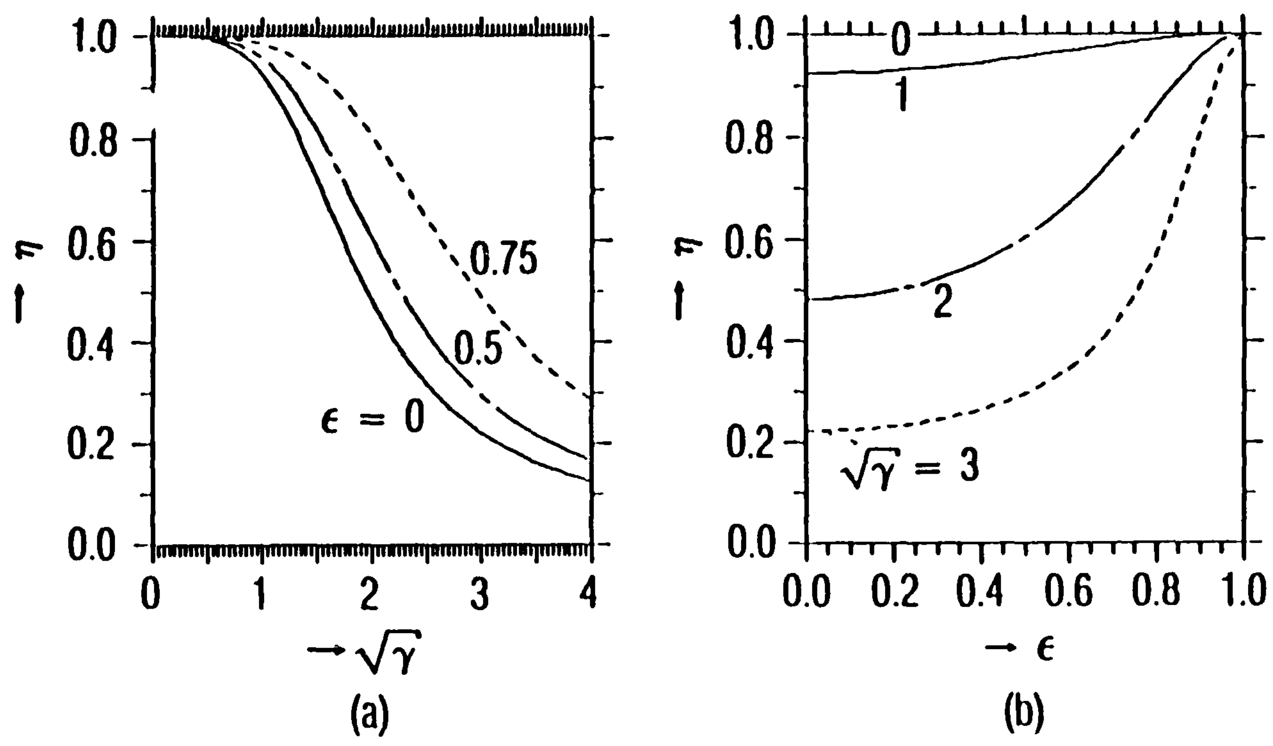
The decrease in η due to an increase in γ can be due to an increase in a and/or a decrease in ω. If we consider the absolute (unnormalized) values of the focal-point irradiance, we note that $I_u(0;R)$ increases quadratically with a, but $I_g(0;R)$ increases only slightly ($\leq$ 10%) with a for $\sqrt{\gamma} \geq 3$. Similarly, $I_g(0;R)$ decreases (nearly) quadratically as ω decreases for $\sqrt{\gamma} \geq 3$. It is evident that η is always less than 1. A redistribution of the aperture power from a uniform to any nonuniform distribution reduces the focal-point irradiance.[20]

4.2 AXIAL IRRADIANCE

If we let r = 0 in Eq. (11), we obtain the axial irradiance for uniform[17] and Gaussian[11] beams

$$I_u(0;z) = (R/z)^2(1 - \varepsilon^2) \{\sin[(1 - \varepsilon^2)\Phi_o/2]/[(1 - \varepsilon^2)\Phi_o/2]\}^2 \tag{23}$$

[20] Mahajan, V. N., "Luneburg Apodization Problem I," Opt. Lett., 5, 1980, pp. 267-269.



Note: $\sqrt{\gamma} = a/\omega$.

Figure 4. Focal-Point Irradiance Ratio η for Uniform and Gaussian Beams as a Function of γ and ε

and

$$I_g(0;z) = (R/z)^2[2\gamma/(\Phi_o^{\,2} + \gamma^2)]\{\coth[(1 - \varepsilon^2)\gamma]$$

$$- \cos[(1 - \varepsilon^2)\Phi_o]/\sinh[(1 - \varepsilon^2)\gamma]\} \qquad (24)$$

respectively. The quantity Φ_o represents the defocus phase aberration at the edge of a circular aperture. It is given by

$$\Phi_o = \left(\frac{A}{\lambda}\right)\left(\frac{1}{z} - \frac{1}{R}\right) \qquad (25a)$$

$$= \pi N\left(\frac{R}{z} - 1\right) \qquad (25b)$$

where

$$N = a^2/\lambda R \qquad (26)$$

is the Fresnel number of the circular aperture as observed from the focus.

The positions of maxima and minima of axial irradiance are obtained by equating to zero its derivative with respect to z. In the case of a uniform beam, the minima have a value of zero. They are located at z values corresponding to an integral number of waves of defocus as an aberration at the outer edge of the aperture relative to that at its inner edge, i.e., for

$$\Phi_o = 2\pi n/(1 - \varepsilon^2), \quad n = \pm 1, \pm 2, \ldots \qquad (27a)$$

or

$$z/R = \{1 + [2n/N(1 - \varepsilon^2)]\}^{-1} \qquad (27b)$$

The positions of maxima of axial irradiance are given by the solutions of

$$\tan[(1 - \varepsilon^2)\Phi_o/2] = (R/z)(1 - \varepsilon^2)\Phi_o/2, \quad z \neq R \qquad (28)$$

In the case of a Gaussian beam, the positions of minima and maxima of axial irradiance are given by the solutions of

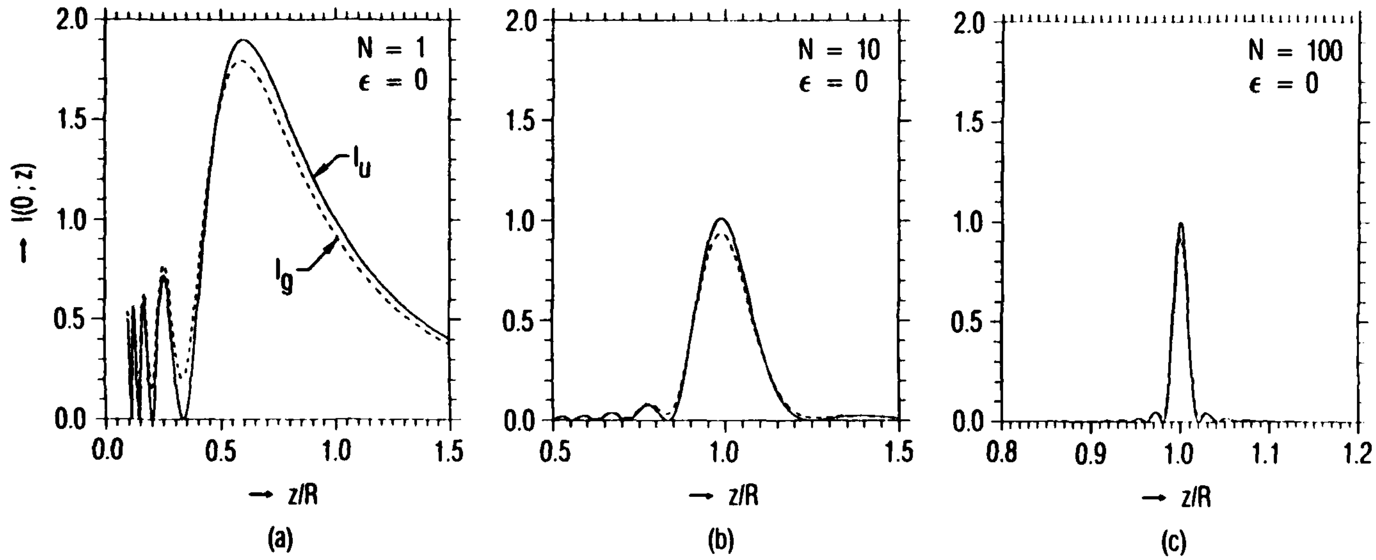
$$2\left(\frac{\lambda z}{A} - \frac{\Phi_o}{\Phi_o^{\,2} + \gamma^2}\right)\{\cosh[(1 - \varepsilon^2)\gamma] - \cos[(1 - \varepsilon^2)\Phi_o]\}$$

$$= -(1 - \varepsilon^2)\sin[(1 - \varepsilon^2)\Phi_o] \qquad (29)$$

Figure 5 shows how the axial irradiance of a uniform focused beam differs from that for a Gaussian beam when $\gamma = 1$ and Fresnel number N = 1, 10, and 100. We note that the principal maximum is higher for the uniform beam compared to that for the Gaussian beam. However, the secondary maxima are higher for the Gaussian beam. Moreover, whereas the axial minima for the uniform beam have a value of zero, the minima for the Gaussian beam have nonzero values. For a given value of ϵ, the locations of maxima and minima, except the principal maximum, are very nearly the same for the two beams. The effect of the obscuration is to reduce the irradiance at the principal maximum but increase it at the secondary maxima. Also, the maxima and minima occur at smaller z values for an annular aperture. These z values correspond approximately to those axial points at which the annular aperture subtends an odd or an even number of Fresnel's halfwave zones, respectively. We note that the curves become symmetric about the focal point z = R as N increases.

Note that even though the principal maximum of axial irradiance does not lie at the focus, maximum central irradiance on a target at a given distance from the aperture is obtained when the beam is focused on it. This can be seen by equating to zero the derivative of axial irradiance, Eqs. (23) and (24), with respect to R. When doing so, the normalization factor $P_o A/\lambda^2 R^2$ should be substituted in these equations with the consequence that the R^2 factor in front of their right-hand side disappears. Figure 6 illustrates how the central irradiance on a target at a fixed distance z varies when the beam is focused at various distances R along the axis. The irradiance in this figure is in units of $P_o A/\lambda^2 z^2$. The quantity $N_z = a^2/\lambda z$ represents the Fresnel number of a circular aperture as observed from the target. As in Figure 2, the maximum irradiance values for uniform and Gaussian ($\gamma = 1$) beams are 1 and 0.924, respectively, when $\epsilon = 0$, and 0.750 and 0.717 when $\epsilon = 0.5$. We note that as N_z increases, the curves become symmetric about R = z.

It is evident from Eqs. (23) and (24) that the axial irradiance depends on z and R through the inverse-square-law dependence in $(R/z)^2$ factor and the defocus aberration Φ_o. The irradiance is symmetric with respect to the sign of Φ_o. We note from Figures 5 and 6 that as the Fresnel number N or N_z becomes large, the axial irradiance becomes symmetric about the point z = R. The reason for this is simple, as may be seen by an examination of Eq. (25).



Note: The irradiance is in units of the focal-point irradiance P_oA/λ^2R^2 for a uniform circular beam. For a uniform beam, the minima of axial irradiance are located at $z/R = 1/3, 1/5, 1/7, \ldots$, when $\varepsilon = 0$ and at $z/R = 3/11, 3/19, 3/27, \ldots$, when $\varepsilon = 0.5$. The Gaussian beam results in this figure are for $\gamma = 1$.

Figure 5. Axial Irradiance of a Beam Focused at a Fixed Distance R with a Fresnel Number N = 1, 10, and 100

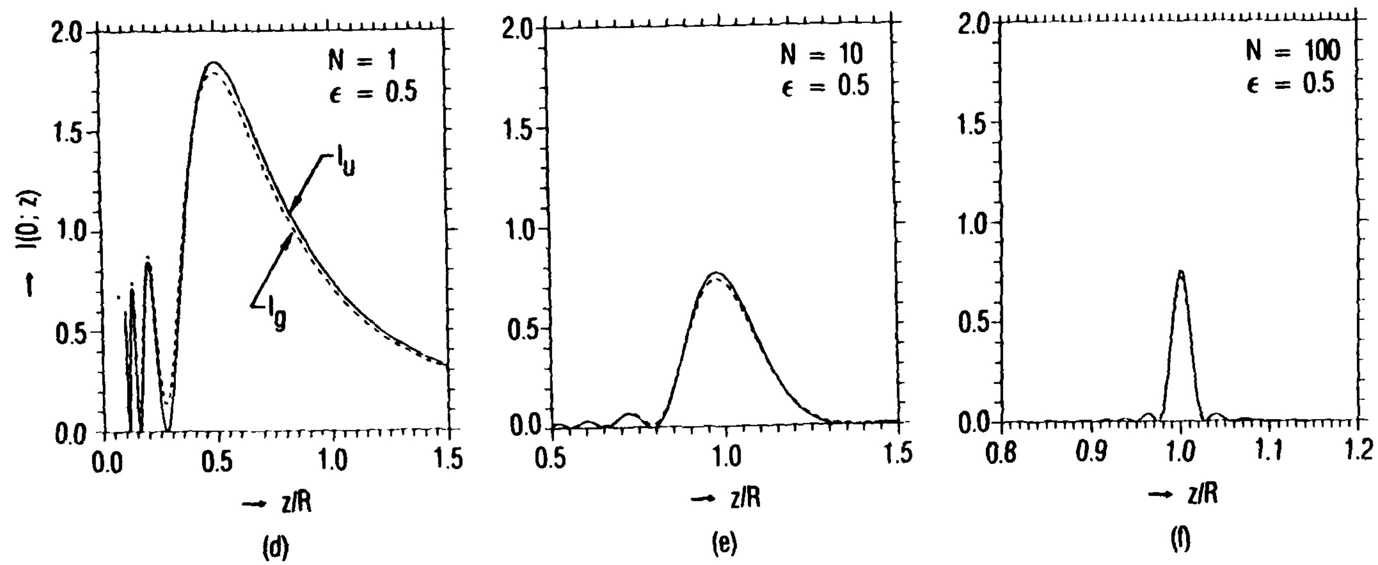


Figure 5. Axial Irradiance of a Beam Focused at a Fixed Distance R with a Fresnel Number N = 1, 10, and 100 (Continued)

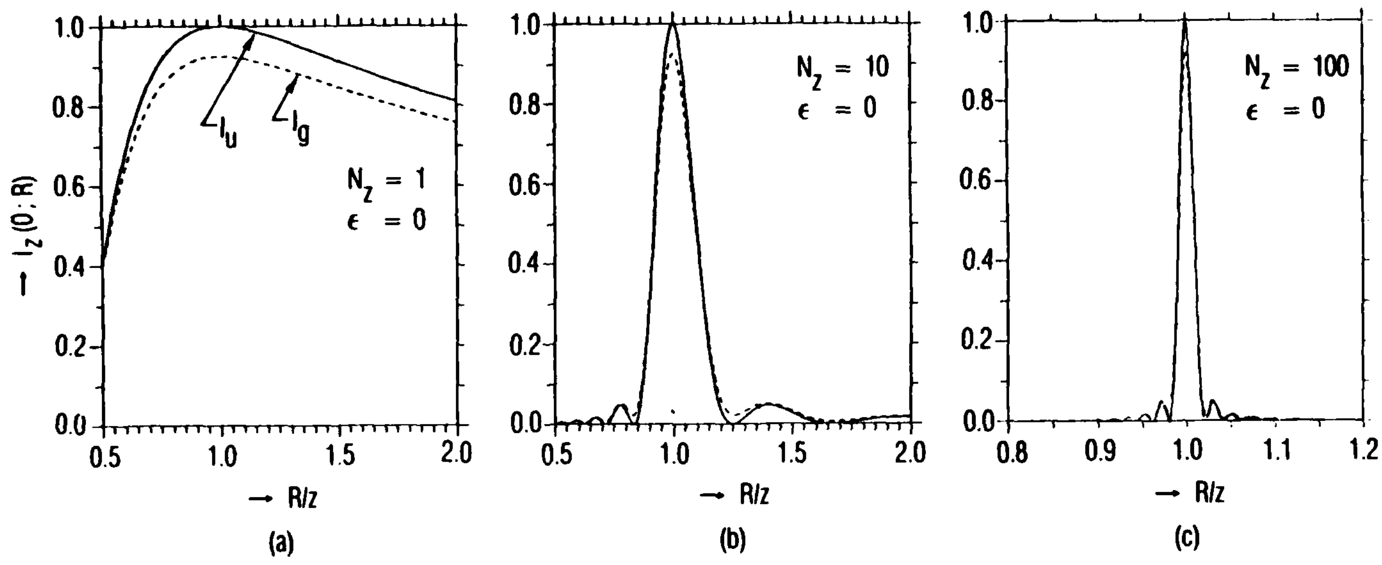


Figure 6. Central Irradiance on a Target at Fixed Distance z from the Aperture Plane When Beam is Focused at Various Distances R

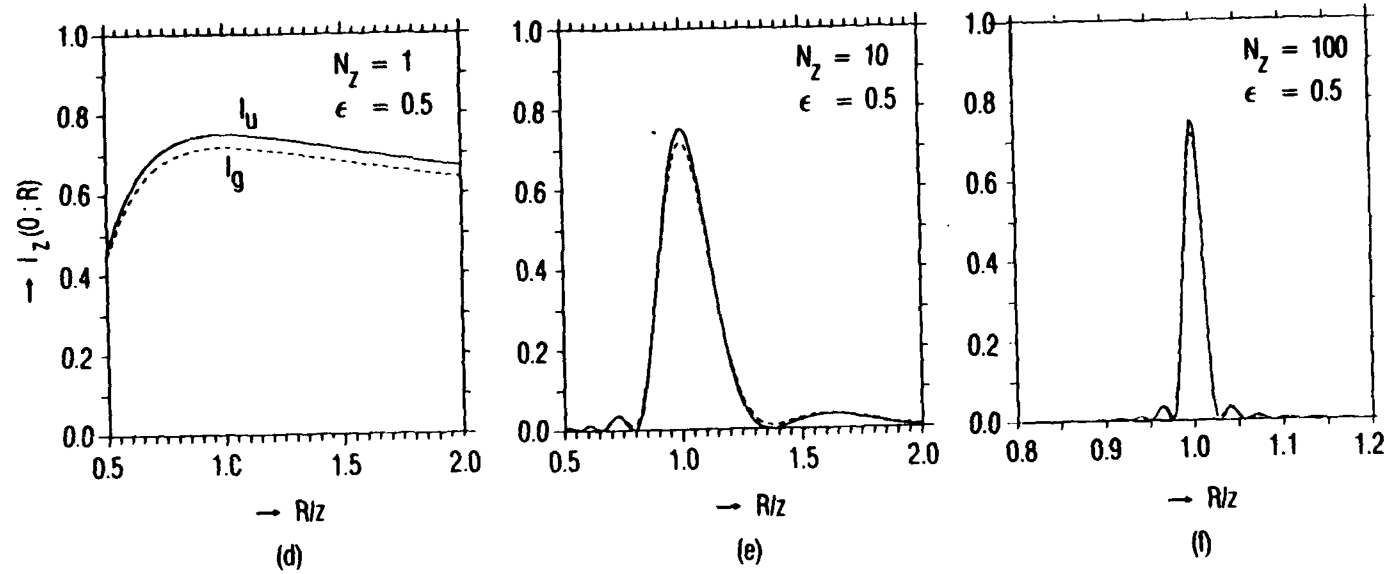


Figure 6. Central Irradiance on a Target at Fixed Distance z from the Aperture Plane When Beam is Focused at Various Distances R (Continued)

When N is very small, z must be much different from R for Φ_o to have a significant value. For example, when N = 1, a quarter wave of defocus (Φ_o = $\pm \pi/2$) is obtained for $R/z = 1 \pm 0.5$. Similarly, when $N_z = 1$, $\Phi_o = \pm \pi/2$ for $z/R = 1 + 0.5$. In either case, the axial irradiance is asymmetric about the point z = R because of the asymmetry in the defocus aberration as well as in the inverse-square-law dependence. However, when N or N_z is very large, a slight difference between z and R gives a large value of Φ_o. For example, when N = 100, a quarter wave of defocus is obtained for $R/z = 1 \pm 0.005$, or $z/R \simeq 1 + 0.005$. Note that, for large N, $|\Phi_o|$ is symmetric about the focal point. Hence, in the vicinity of focus, where the axial irradiance is appreciable compared to the focal-point irradiance, the axial irradiance is symmetric about the focal point. The inverse-square-law dependence has negligible effect in this case, since z and R are practically equal to each other. Similar considerations hold when N_z is very large, i.e., the irradiance at the point of observation is the same for two beams focused symmetrically about it. These considerations also extend to the transverse irradiance distributions [see Eq. (30)]. Thus, for example, the transverse irradiance distribution in the vicinity of the focal plane is symmetric about it only when N is very large.

4.3 DEFOCUSED DISTRIBUTION

When $z \neq R$, we may write the right-hand side of Eq. (11) as the product of two integrals and retain only its real part, since irradiance is a real quantity. Thus, the irradiance distribution in a defocused plane can be written

$$I(r;z) = 4(R/z)^2 \int_\epsilon^1 \int_\epsilon^1 \rho s \sqrt{I(\rho)I(s)} \cos[\Phi_o(\rho^2 - s^2)] J_0(\pi r\rho) J_0(\pi r s) d\rho ds \qquad (30)$$

If we let r = 0 and note that $J_0(0) = 1$, we obtain a different form of the expression for axial irradiance, namely

$$I(0;z) = 4(R/z)^2 \int_\epsilon^1 \int_\epsilon^1 \rho s \sqrt{I(\rho)I(s)} \cos[\Phi_o(\rho^2 - s^2)] d\rho ds \qquad (31)$$

The encircled power (in units of P_o with r_o in units of $\lambda z/D$) is given by

$$P(r_o;z) = 2\pi^2 \int_\varepsilon^1 \int_\varepsilon^1 \rho s\sqrt{I(\rho)I(s)} \cos[\Phi_o(\rho^2 - s^2)]Q(\rho,s;r_o)d\rho ds \qquad (32)$$

where[21]

$$Q(\rho,s;r_o) = \int_0^{r_o} J_0(\pi r\rho)\, J_0(\pi rs)rdr$$

$$= (r_o^2/2)[J_0^2(\pi r_o\rho) + J_1^2(\pi r_o s)] \text{ if } \rho = s \qquad (33a)$$

$$= [r_o/\pi(\rho^2 - s^2)][\rho J_1(\pi r_o\rho)\, J_0(\pi r_o s)$$

$$-sJ_1(\pi r_o s)\, J_0(\pi r_o\rho)] \text{ if } \rho \neq s \qquad (33b)$$

The integrals in Eqs. (30)-(32) can be evaluated by the Gauss quadrature method according to which we may write[22]

$$\int_\varepsilon^1\int_\varepsilon^1 f(\rho,s)\, d\rho\, ds = [(1 - \varepsilon)/2]^2 \left[\sum_{i=1}^{M} \omega_i^2\, f(\rho_i,s_i) \right.$$

$$\left. + \; 2\sum_{i=2}^{M} \sum_{j=1}^{i-1} \omega_i\omega_j\, f(\rho_i,s_j) \right] \qquad (34)$$

where M is the number of 1-D quadrature points, ω_i are the weight factors, and

$$\rho_i = s_i$$

$$= [1 + \varepsilon + (1 - \varepsilon)x_i]/2 \qquad (35)$$

x_i being the ith zero of the Mth-order Legendre polynomial. In our calculations we have used a 24-point (M = 24) Gauss quadrature. Note that by letting

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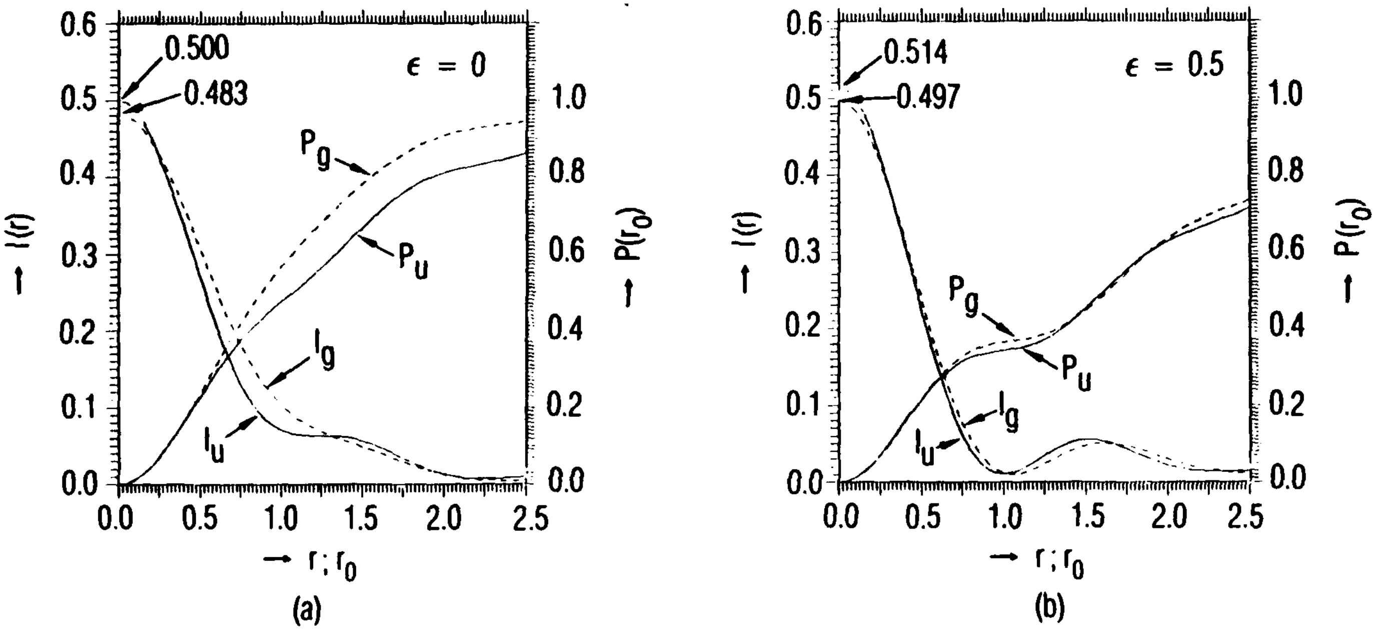
[21] Abramowitz, M., and I. A. Stegun, Handbook of Mathematical Functions, Dover, NY, 1970, p. 484.

[22] Ibid., p. 887.

$\Phi_o = 0$ in Eqs. (30)-(32), we can calculate the focal-plane distributions as well. Equations (30)-(35) are generalizations of our earlier work on uniform circular beams[23] to annular apertures and Gaussian beam. Note that with slight modification, Eqs. (30)-(32) can be applied to diffraction calculations involving any radially symmetric amplitude and phase distributions at the aperture. For example, if spherical aberration ($A_s\rho^4$) were present, the cosine factor in these equations would become $\cos[\Phi(\rho) - \Phi(s)]$ where $\Phi(\rho) = \Phi_o\rho^2 + A_s\rho^4$.

An example of defocused distribution is illustrated in Figure 7 for both uniform and Gaussian ($\gamma = 1$) beams with a large Fresnel number (so that the inverse-square-law variation is negligible) when $\epsilon = 0$ and $\epsilon = 0.5$. The amount of defocus $\Phi_o = 2.783$ rad (or 0.443λ) is such that the central irradiance for a uniform circular beam is reduced to half the corresponding focal-point irradiance. (The defocused distributions shown can also be interpreted as the distributions on a target at a fixed distance z when the beam is focused at a distance R such that $\Phi_o = 2.783$ rad. In this case, the irradiance would be in units of P_oA/λ^2z^2 and r and r_o would be in units of $\lambda z/D$.) We note that, as in the case of focal-plane distributions, the central irradiance for a Gaussian beam is lower than that for a corresponding uniform beam. Note, however, that the defocus aberration does not reduce the central irradiance for the annular beam as much as it does for the circular beam, so much so that, for the amount of defocus aberration considered in Figure 7, the defocused central irradiance for the annular beam is higher than that for the corresponding circular beam. For the uniform and Gaussian circular beams, the central irradiance decreases from 1 and 0.924 to 0.500 and 0.483, respectively. For the annular beams, it decreases from 0.750 and 0.717 to 0.514 and 0.497, respectively. This indicates the well-known fact that the tolerance for a radially symmetric aberration such as defocus is higher for an annular beam than that for a circular beam. When $\epsilon = 0$, the encircled power is higher for a uniform beam for small values of r_o compared to that for a Gaussian beam. When $\epsilon = 0.5$, the difference in encircled power for the two types of beam changes from positive to negative to positive depending on the value of r_o.

[23] Mahajan, V. N., "Aberrated Point-Spread Function for Rotationally Symmetric Aberrations," Appl. Opt., 22, 1983, pp. 3035-3041.



Note: The amount of defocus aberration Φ_o = 2.783 rad (0.443λ) is such that it gives a central irradiance of 0.5 for a uniform circular beam of large Fresnel number. The units of r, r_o, I(r), and $P(r_o)$ are the same as in Figure 3.

Figure 7. Defocused Irradiance and Encircled-Power Distributions for Uniform and Gaussian (γ = 1) Beams

5. COLLIMATED BEAM

The results for a collimated beam can be obtained from those for a focused beam by letting $R \to \infty$. Thus, for example, Eq. (11) for the irradiance distribution in a plane at a distance z from the aperture reduces to

$$I(r;z) = 4\Phi_o^2 \left| \int_\varepsilon^1 \sqrt{I(\rho)} \exp[i\Phi_o \rho^2] J_0(\pi r\rho)\rho d\rho \right|^2 \tag{36}$$

where

$$\Phi_o = A/\lambda z \tag{37}$$

represents the (negative of) phase aberration of a plane wavefront with respect to a reference sphere centered at a distance z and passing through the center of the aperture. In Eq. (36), the irradiance in both the aperture and the observation planes is in units of the aperture irradiance P_o/A for a uniform circular beam. As in Eq. (11), r is in units of $\lambda z/D$.

In the far field, i.e., for $z \geq D^2/\lambda$, the phase aberration $\Phi_o \leq \pi/4$ (corresponding to a wave aberration of less than or equal to $\lambda/8$) and may be neglected. Hence, the irradiance distribution, and correspondingl the encircled-power distribution in a far-field plane, is similar to a focal-plane distribution discussed earlier. The only difference is in scaling of the diffraction pattern. Similarly, in the near-field, i.e., for $z < D^2/\lambda$, the irradiance and encircled-power distributions correspond to defocused distributions discussed earlier. The only significant difference is in the definition of Φ_o.

If z is in units of the far-field distance D^2/λ, and we let $r = 0$ in Eq. (36), we obtain the axial irradiance (in units of P_o/A) for uniform[17] and Gaussian beams

$$I_u(0;z) = [4/(1 - \varepsilon^2)] \sin^2[\pi(1 - \varepsilon^2)/8z] \tag{38}$$

and

$$I_g(0;z) = \{2\gamma/[1 + (4\gamma z/\pi)^2]\} \{\coth[\gamma(1 - \varepsilon^2)]$$

$$- \cos[\pi(1 - \varepsilon^2)/4z]/\sinh[\gamma(1 - \varepsilon^2)]\} \tag{39}$$

respectively. For a uniform beam, the maxima of axial irradiance have a value of $4/(1 - \varepsilon^2)$ at

$$z = (1 - \varepsilon^2)/4(2n + 1) , \qquad n = 0, 1, 2, \ldots \tag{40}$$

Its minima have a value of zero at

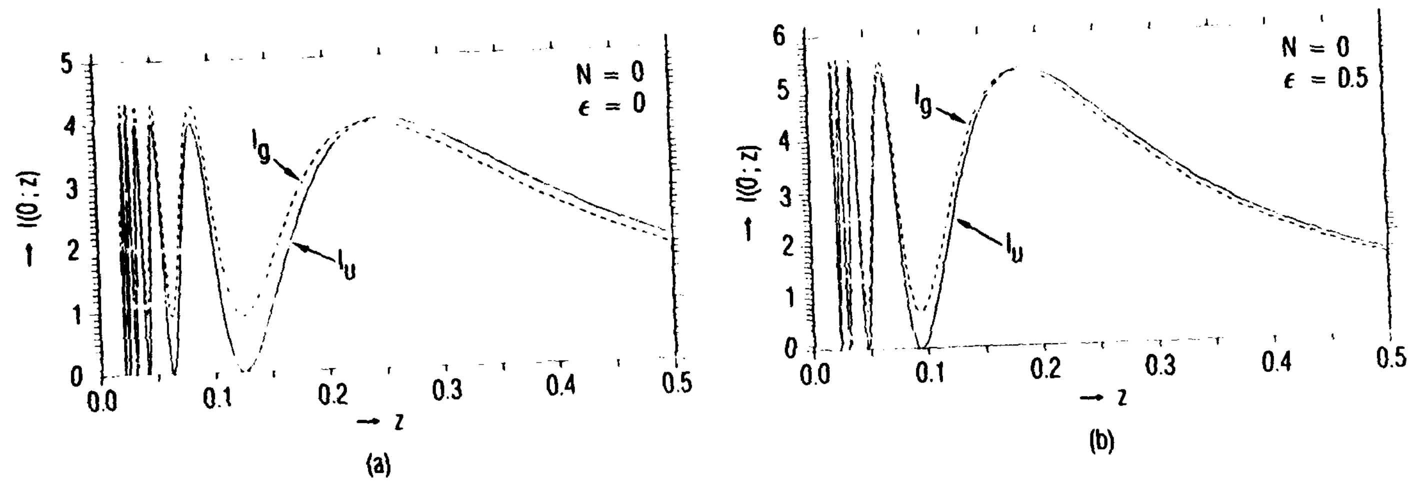
$$z = (1 - \varepsilon^2)/8n , \qquad n = 1, 2, \ldots \tag{41}$$

The positions of maxima and minima of axial irradiance for a Gaussian beam are given by an appropriate modification of Eq. (29), namely

$$\{2(4z/\pi)^3\gamma^2/[1 + (4\gamma z/\pi)^2]\}\{\cosh[\gamma(1 - \varepsilon^2)] - \cos[\pi(1 - \varepsilon^2)/4z]\}$$

$$= - (1 - \varepsilon^2)\sin[\pi(1 - \varepsilon^2)/4z] \tag{42}$$

Figure 8 illustrates how the axial irradiance of collimated uniform and Gaussian beams varies with distance z from the aperture. With reference to Figure 5, Figure 8 corresponds to N = 0. In contrast to Figure 5, the maxima of axial irradiance of a collimated uniform beam have the same value of $4/(1 - \varepsilon^2)$. Moreover, unlike the principal maximum in Figures 5a and 5d, the maximum farthest from the aperture has a lower value than the maxima closer to it in the case of a Gaussian beam.



Note: The irradiance is in units of the uniform irradiance P_o/A at the aperture of a uniform circular beam. The distance z is in units of the far-field distance D^2/λ for a uniform circular aperture.

Figure 8. Axial Irradiance of Collimated Uniform and Gaussian Beams

6. ABERRATION BALANCING

A focused beam emerging from the aperture is aberration free, if its wavefront passing through the center of the aperture is spherical with a center of curvature at the focus. Any deviations of the wavefront from this spherical form represent aberrations. In that case, the spherical wavefront centered at the focus is referred to as the reference sphere. For small aberrations, the Strehl ratio (i.e., the ratio of central irradiances with and without an aberration) is determined by the aberration variance across the amplitude-weighted aperture.[24] The problem of aberration balancing for uniformly illuminated circular apertures was discussed by Nijboer.[25,26] This has been extended to nonuniformly illuminated apertures.[24,27] In aberration balancing, a classical aberration of a certain order (which represents a term in the power series expansion of the aberration function in aperture coordinates) is mixed with aberrations of lower order such that the variance of the net aberration is minimized. Consider, for example, a typical balanced aberration[24] (representing a term in the expansion of the aberration in terms of a set of "Zernike" polynomials which are orthonormal over the amplitude-weighted annular aperture)

$$\Phi(\rho,\theta;\varepsilon) = c_{nm}\,\varepsilon_m \sqrt{2(n+1)}\, R_n^m(\rho;\varepsilon)\cos m\theta, \tag{43}$$

where (ρ,θ) are the polar coordinates of a point in the aperture plane, n and m are positive integers (including zero), $n - m \geq 0$ and even, $R_n^m(\rho;\varepsilon)$ is a radial polynomial of degree n in ρ and has the form

$$R_n^m(\rho;\varepsilon) = a_n^m \rho^n + b_n^m \rho^{n-2} + \ldots + d_n^m \rho^m \tag{44}$$

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24 Mahajan, V. N., "Zernike Annular Polynomials for Imaging Systems with Annular Pupils," J. Opt. Soc. Am., 71, 1981, pp. 75-85, 1408.

25 Nijboer, B. R. A., "The Diffraction Theory of Aberrations," Ph.D. Thesis (University of Groningen, Groningen, The Netherlands, 1942).

26 See Ref. 18, Chapter 9.

27 Szapiel, S., "Aberration Balancing Technique for Radially Symmetric Amplitude Distributions: A Generalization of the Marechal Approach," J. Opt. Soc. Am., 72, 1982, pp. 947-956.

The coefficients a_n^m et cetera, depend on the obscuration ratio ε and the aperture amplitude $\sqrt{I(\rho)}$. The quantity

$$\varepsilon_m = \frac{1}{\sqrt{2}}, \quad m = 0$$
$$= 1, \quad m \neq 0 \tag{45}$$

Unless $n = m = 0$, the coefficient c_{nm} represents the standard deviation of the aberration, i.e.

$$c_{nm} = \sigma_\Phi \tag{46}$$

where

$$\sigma_\Phi^2 = \langle\Phi^2\rangle - \langle\Phi\rangle^2 \tag{47}$$

and

$$\langle\Phi^n\rangle = \int_\varepsilon^1 \int_0^{2\pi} \sqrt{I(\rho)}\ \Phi^n(\rho,\theta)\rho d\rho d\theta \Bigg/ \int_\varepsilon^1 \int_0^{2\pi} \sqrt{I(\rho)}\ \rho d\rho d\theta \tag{48}$$

From Eqs. (43) and (46), we note that the standard deviation of an aberration can be obtained immediately by comparing its form with the corresponding orthonormal aberration represented by Eq. (43), without having to calculate the integrals in Eq. (48). The variance of an aberration consisting of two or more terms of the form of Eq. (43) is given by the sum of the variance of each of the aberration terms.

The radial polynomials corresponding to balanced primary aberrations are listed in Table 2 for both uniform and Gaussian beams.[24] We now consider spherical aberration, coma, and astigmatism in uniform and Gaussian beams and compare their balancing with other aberrations to minimize their variance across the aperture.

6.1 SPHERICAL ABERRATION

First, we consider spherical aberration

$$\Phi_s(\rho) = A_s\rho^4 \tag{49}$$

Table 2. Radial Polynomials for Balanced Primary Aberrations for Uniform and Gaussian Beams

Aberration	Polynomial	Uniform	Gaussian	Weakly Truncated Gaussian ($\epsilon = 0$)
Piston	R_0^0	1	1	1
Distortion (tilt)	R_1^1	$\rho/(1+\epsilon^2)^{1/2}$	$a_1^1\rho$	$\sqrt{\gamma/2}\,\rho$
Field Curvature (defocus)	R_2^0	$(2\rho^2 - 1 - \epsilon^2)/(1-\epsilon^2)$	$a_2^0\rho^2 + b_2^0$	$(\gamma\rho^2 - 1)/\sqrt{3}$
Astigmatism	R_2^2	$\rho^2/(1+\epsilon^2+\epsilon^4)^{1/2}$	$a_2^2\rho^2$	$(\gamma/\sqrt{6})\rho^2$
Coma	R_3^1	$\dfrac{3(1+\epsilon^2)\rho^3 - 2(1+\epsilon^2+\epsilon^4)\rho}{(1-\epsilon^2)[(1+\epsilon^2)(1+4\epsilon^2+\epsilon^4)]^{1/2}}$	$a_3^1\rho^3 + b_3^1\rho$	$\sqrt{\gamma/2}\,(\frac{\gamma}{2}\rho^3 - \rho)$
Spherical Aberration	R_4^0	$[6\rho^4 - 6(1+\epsilon^2)\rho^2 + (1+4\epsilon^2+\epsilon^4)]/(1-\epsilon^2)^2$	$a_4^0\rho^4 + b_4^0\rho^2 + c_4^0$	$(\gamma^2\rho^4 - 4\gamma\rho^2 + 2)/2\sqrt{5}$

$a_1^1 = (2p_2)^{-1/2}$, $a_2^0 = [3(p_4 - p_2^2)]^{-1/2}$, $b_2^0 = -p_2 a_2^0$, $a_2^2 = (3p_4)^{-1/2}$, $a_3^1 = \frac{1}{2}(p_6 - p_4^2/p_2)^{-1/2}$, $b_3^1 = -(p_4/p_2)a_3^1$,

$a_4^0 = \{5[p_8 - 2K_1 p_6 + (K_1^2 + 2K_2)p_4 - 2K_1K_2p_2 + K_2^2]\}^{-1/2}$, $b_4^0 = -K_1 a_4^0$, $c_4^0 = K_2 a_4^0$

$p_s = \langle\rho^s\rangle = [(\epsilon^s e^{\gamma(1-\epsilon^2)} - 1)/(e^{\gamma(1-\epsilon^2)} - 1)] + (s/2\gamma)p_{s-2}$, s is an even integer, $p_0 = 1$

$K_1 = (p_6 - p_2p_4)/(p_4 - p_2^2)$, $K_2 = (p_2p_6 - p_4^2)/(p_4 - p_2^2)$

where A_s represents the peak value of the aberration. For a uniform beam, the standard deviation of the aberration is given by[24]

$$\sigma_{su} = [(4 - \varepsilon^2 - 6\varepsilon^4 - \varepsilon^6 + 4\varepsilon^8)^{1/2}/3\sqrt{5}]A_s \tag{50}$$

If we mix the aberration with an appropriate amount of defocus $A_d\rho^2$, we obtain the balanced aberration

$$\Phi_{bs}(\rho) = A_s\rho^4 + A_d\rho^2 \tag{51}$$

The value of A_d which minimizes the variance of the balanced aberration can be obtained by comparing it with the radial polynomial $R_4^0(\rho;\varepsilon)$ given in Table 2. For a uniform beam it is given by

$$A_{du} = -(1 + \varepsilon^2)A_s \tag{52}$$

The corresponding standard deviation of the balanced aberration is given by[24]

$$\sigma_{bsu} = [(1 - \varepsilon^2)^2/6\sqrt{5}]A_s \tag{53}$$

Note that for a given value of A_s, σ_{bsu} decreases as ε increases.

For a Gaussian circular beam with $\gamma = 1$

$$\sigma_{sg} = [(20e^2 - 69e + 40)^{1/2}/(e - 1)]A_s \tag{54a}$$

$$= A_s/3.668 \tag{54b}$$

The balancing defocus and the standard deviation of the corresponding balanced aberration are given by

$$A_{dg} = (b_4^0/a_4^0)A_s \tag{55a}$$

$$= -0.933\ A_s \tag{55b}$$

and

$$\sigma_{bsg} = A_s/\sqrt{5}\ a_4^0 \tag{56a}$$

$$= A_s/13.705 \tag{56b}$$

Comparing uniform and Gaussian circular beams aberrated by spherical aberration, we note the following. For a given value of A_s

$$\sigma_{sg}/\sigma_{su} = 0.91 \tag{57}$$

Hence, for a given Strehl ratio $\exp(-\sigma^2)$, where σ^2 is the variance of the phase aberration,[28] a Gaussian beam can tolerate a slightly higher amount of spherical aberration than a uniform beam. By balancing with an appropriate amount of defocus, the standard deviation for a Gaussian beam is reduced by a factor of 3.74 compared to a factor of 4 for a uniform beam. Comparing the standard deviation of the balanced aberration for the two beams, we find that

$$\sigma_{bsg}/\sigma_{bsu} = 0.98 \tag{58}$$

Hence, for a given value of A_s, the Strehl ratios for the two beams are practically the same. We noted earlier that the central irradiance for an aberration-free Gaussian beam is lower by 7.6% compared to that for a corresponding unifor- beam. For beams aberrated with a small value of A_s, the difference in their peak central irradiances will be slightly smaller. Comparing the amounts of balancing defocus

$$A_{dg}/A_{du} = 0.933 \tag{59}$$

we note that the defocused plane for a Gaussian beam is closer to the focal plane than that for a uniform beam. The location of the defocused plane, i.e., its z value, is given by

$$\Phi_o = A_d \tag{60}$$

For $z \simeq R$, it is given by

$$z = R - 8\lambda F^2 A_d \tag{61}$$

where A_d is in units of λ.

To compare the central irradiance in the focal plane, we note that for a quarter wave of spherical aberration, a Strehl ratio of 0.8 is obtained for a uniform beam[28] corresponding to $\sigma_u{}^2 = 0.22$. The corresponding Strehl ratio for a Gaussian beam is $e^{-0.22(0.91)^2} = 0.83$, giving a central irradiance of 0.77 (compared to 0.8 for a uniform beam).

6.2 COMA

Next we consider coma

$$\Phi_c(\rho,\theta) = A_c\rho^3 \cos\theta \tag{62}$$

where A_c represents the peak value of the aberration. For a uniform beam, the standard deviation of the aberration is given by[24]

$$\sigma_{cu} = [(1 + \varepsilon^2 + \varepsilon^4 + \varepsilon^6)/8]^{1/2} A_c \tag{63}$$

If we mix coma with an appropriate amount of tilt, $A_t\rho \cos\theta$, we obtain the balanced aberration

$$\Phi_{bc}(\rho,\theta) = (A_c\rho^3 + A_t\rho) \cos\theta \tag{64}$$

The value of A_t, which minimizes the variance of the balanced aberration, can be obtained by comparing it with the radial polynomial $R_3^1(\rho;\varepsilon)$. For a uniform beam, it is given by

$$A_{tu} = -(2/3)[(1 + \varepsilon^2 + \varepsilon^4)/(1 + \varepsilon^2)]A_c \tag{65}$$

The corresponding standard deviation of the balanced aberration is given by[24]

$$\sigma_{bcu} = \frac{(1 - \varepsilon^2)(1 + 4\varepsilon^2 + \varepsilon^4)^{1/2}}{6\sqrt{2}\,(1 + \varepsilon^2)^{1/2}} A_c \tag{66}$$

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[28] Mahajan, V. N., "Strehl Ratio for Primary Aberrations in Terms of Their Aberration Variance," J. Opt. Soc. Am., 73, 1983, pp. 860-861.

For a given value of A_c, σ_{bcu} slightly increases as ε increases, achieves its maximum value at $\varepsilon = 0.29$, and decreases rapidly for larger values of ε.

For a Gaussian circular beam with $\gamma = 1$

$$A_{tg} = (b_3^1/a_3^1)A_c \tag{67a}$$

$$= -0.608\ A_c \tag{67b}$$

and

$$\sigma_{bcg} = A_c/2\sqrt{2}\ a_3^1 \tag{68a}$$

$$= A_c/8.802 \tag{68b}$$

Comparing uniform and Gaussian circular beams aberrated by coma, we note the following. For a given value of A_c

$$\sigma_{bcg}/\sigma_{bcu} = 0.96 \tag{69}$$

Therefore, the Strehl ratio for a Gaussian beam is approximately 0.82 when it is 0.80 for a uniform beam. Accordingly, the peak irradiances for the two beams are 0.75 and 0.80, respectively. Note, however, that since

$$A_{tg}/A_{tu} = 0.91 \tag{70}$$

the peak irradiance for a Gaussian beam occurs in the focal plane at a point which is closer to the focal point by 9% compared to that for a uniform beam.

6.3 ASTIGMATISM

Finally, we consider astigmatism

$$\Phi_a(\rho, \theta) = A_a\rho^2 \cos^2\theta \tag{71}$$

where A_a represents the peak value of the aberration. For a uniform beam, the standard derivation of the aberration is given by

$$\sigma_{au} = (1/4)(1 + \varepsilon^4)^{1/2} A_a \tag{72}$$

If we mix astigmatism with an appropriate amount of defocus, we obtain the balanced aberration

$$\Phi_{ba}(\rho,\theta) = A_a\rho^2 \cos^2\theta + A_d\rho^2 \tag{73}$$

For both uniform and Gaussian beams, the amount of defocus that minimizes the variance of the balanced aberration can be obtained by comparing it with the Zernike polynomial $R_2^2(\rho;\varepsilon)\cos 2\theta$. It is given by

$$A_d = -(1/2)A_a \tag{74}$$

so that the balanced aberration becomes

$$\Phi_{ba}(\rho,\theta) = (1/2)A_a\rho^2 \cos 2\theta \tag{75}$$

For a uniform beam, the corresponding standard deviation is given by

$$\sigma_{bau} = (1/2\sqrt{6})(1 + \varepsilon^2 + \varepsilon^4)^{1/2} A_a \tag{76}$$

For a given value of A_a, σ_{au} and σ_{bau} both increase as ε increases. This is true for a Gaussian beam as well. For a Gaussian circular beam with $\gamma = 1$

$$\sigma_{bag} = A_a/2\sqrt{6}\ a_2^2 \tag{77a}$$

$$= A_a/5.609 \tag{77b}$$

Comparing uniform and Gaussian circular beams, we note that, for given value of A_a

$$\sigma_{bag}/\sigma_{bau} = 0.87 \tag{78}$$

Therefore, the Strehl ratio for a Gaussian beam is approximately 0.85 compared to a value of 0.80 for a uniform beam. Accordingly, the central irradiances for the two beams are 0.78 and 0.80, respectively.

For easy comparison, the standard deviation of primary aberrations and of the corresponding balanced aberrations is tabulated in Table 3 for uniform and Gaussian ($\gamma = 1$) circular beams. It is evident that the standard deviation for a Gaussian beam is somewhat smaller than the corresponding value for a uniform beam. Accordingly, for a given small amount of aberration, the Strehl ratio for a Gaussian beam is higher than the corresponding value for a uniform beam. Similarly, for a given Strehl ratio, the aberration tolerance for a Gaussian beam is somewhat higher than that for a uniform beam. We also note that, whereas aberration balancing in the case of a uniform beam reduces the standard deviation of spherical aberration and coma by factors of 4 and 3, respectively, the reduction in the case of astigmatism is only a factor of 1.22. For a Gaussian ($\gamma = 1$) beam, the reduction factors have a similar trend, but the factors are smaller: 3.74, 2.64, and 1.16 corresponding to spherical aberration, coma, and astigmatism, respectively.

We note that the variance of each of the three primary aberrations is minimized if it is measured with respect to a reference sphere centered at a point which is slightly different from the focal point. In the case of spherical aberration and astigmatism, the balancing aberration is defocus, i.e., their variance is minimized if they are measured with respect to a reference sphere centered on an axial point $z \neq R$. In the case of coma, the balancing aberration is tilt; i.e., its variance is minimized if it is measured with respect to a reference sphere centered at a point in the focal plane but not at the focal point. For small aberrations, minimum variance has the consequence that when an aberrated beam is focused on a target at a fixed distance z from the aperture, the central irradiance on it is maximum if the beam is slightly defocused in the case of spherical aberration and astigmatism and if it is slightly tilted in the case of coma. If the beam is neither defocused nor tilted, then we can say that for focusing systems with large Fresnel numbers (so that the highest aberration-free irradiance peak occurs at the focal point), the highest peak of the three-dimensional irradiance distribution occurs at the point with respect to which the aberration variance is minimum.

Table 3. Standard Deviation of Primary Aberrations and of the Corresponding Balanced Aberrations for Uniform and Gaussian ($\gamma = 1$) Circular Beams

Aberration, $\Phi(\rho,\theta)$	Standard Deviation	
	Uniform ($\gamma = 0$)	Gaussian ($\gamma = 1$)
Spherical		
$A_s\rho^4$	$A_s/3.35$	$A_s/3.67$
Balanced Spherical		
$A_s\rho^4 + A_d\rho^2$	$A_s/13.42$	$A_s/13.71$
Coma		
$A_c\rho^3\cos\theta$	$A_c/2.83$	$A_c/3.33$
Balanced Coma		
$(A_c\rho^3 + A_t\rho)\cos\theta$	$A_c/8.49$	$A_c/8.80$
Astigmatism		
$A_a\rho^2\cos^2\theta$	$A_a/4$	$A_a/4.84$
Balanced Astigmatism		
$A_a\rho^2\cos^2\theta + A_d\rho^2 = (1/2)A_a\cos^2\theta$	$A_a/4.90$	$A_a/5.61$

$A_{du} = -A_s$, $A_{dg} = -0.93A_s$; $A_{tu} = -(2/3)A_c$, $A_{tg} = -0.61A_c$;

$A_{du} = A_{dg} = -A_a/2$

7. WEAKLY TRUNCATED GAUSSIAN CIRCULAR BEAMS

7.1 IRRADIANCE DISTRIBUTION AND BEAM RADIUS

In this section, we consider a weakly truncated Gaussian circular beam, i.e., one for which the aperture radius a is much larger than the beam radius ω. When $a >> \omega$, i.e., when γ is very large, $f(\gamma;0) \to 2\gamma$; therefore, Eq. (5) for the aperture irradiance distribution may be written

$$I(\rho) = 2\gamma \exp(-2\gamma\rho^2) \tag{79a}$$

or

$$I(\rho) = (2P_o/\pi\omega^2) \exp[-2(\rho/\omega)^2] \tag{79b}$$

In Eq. (79a), $I(\rho)$ is in units of P_o/A and ρ is in units of a as stipulated in Eq. (11). In Eq. (79b), no such normalization is used.

For large γ, the upper limit of integration in Eq. (11) may be replaced by infinity with a negligible error. Since we are considering a circular beam, the lower limit ε is zero. Hence, if we let $\beta = \pi r$ and $\alpha = \gamma - i\Phi_o$, the integral in Eq. (11) reduces to the form

$$\int_0^\infty \exp(-\alpha\rho^2) J_0(\beta\rho)\rho d\rho \tag{80a}$$

which for $Re\alpha > 0$ is equal to[29]

$$(1/2\alpha) \exp(-\beta^2/4\alpha) \tag{80b}$$

Accordingly, it can be shown that, for large γ, Eq. (11) reduces to

$$I(r;z) = (R/z)^2[2\gamma/(\Phi_o^2 + \gamma^2)]\exp[-\gamma\pi^2 r^2/2(\Phi_o^2 + \gamma^2)] \tag{81a}$$

[29]Gradshteyn, I. S., and I. M. Ryzhik, Tables of Integrals, Series, and Products, 4th ed., Academic, NY, 1965, p. 717.

or

$$I(r;z) = (2P_o/\pi\omega_z^2)\ \exp(-2r^2/\omega_z^2) \tag{81b}$$

where

$$\omega_z^2 = (\lambda z/\pi\omega)^2 + \omega^2(1 - z/R)^2 \tag{82}$$

In Eq. (81a), the irradiance is in units of P_oA/λ^2R^2 and r is in units of $\lambda z/D$ as was the case in Eq. (11). In Eq. (81b), these quantities are not normalized.

Comparing Eqs. (79) and (81), we note the well-known result that, if the truncation of the beam by the aperture is neglected, a Gaussian beam propagates as a Gaussian beam. The beam radius at a distance z from the aperture is ω_z given by Eq. (82).

If we let r = 0 in Eq. (81), we obtain the axial irradiance

$$I(0;z) = (R/z)^2[2\gamma/(\Phi_o^2 + \gamma^2)] \tag{83a}$$

or

$$I(0;z) = 2P_o/\pi\omega_z^2 \tag{83b}$$

Of course, for large γ and $\epsilon = 0$, Eq. (24) reduces to Eq. (83) as expected.

If we let z = R in Eq. (81), we obtain the focal-plane irradiance distribution

$$I(r;R) = (2/\gamma)\exp(-\pi^2r^2/2\gamma) \tag{84a}$$

or

$$I(r;R) = (2P_o/\pi\omega_R^2)\ \exp(-2r^2/\omega_R^2) \tag{84b}$$

where

$$\omega_R = \lambda R/\pi\omega \tag{84c}$$

is the beam radius in the focal plane. The focal-point irradiance is given by

$$I(0;R) = 2/\gamma \tag{85a}$$

$$I(0;R) = 2\pi P_o \omega^2/\lambda^2 R^2 \tag{85b}$$

a result already obtained in Eq. (22).

If we equate to zero the derivative of Eq. (83) with respect to z, we obtain the z value at which the axial irradiance is maximum. Note that this value is given by

$$z_p/R = [1 + (\gamma/\pi N)^2]^{-1} \tag{86a}$$

or

$$z_p/R = [1 + (\lambda R/\pi\omega^2)^2]^{-1} \tag{86b}$$

Substituting Eq. (86) into Eq. (83), we obtair the peak value of axial irradiance

$$I(0;z_p) = (2/\gamma) + (2\gamma/\pi^2 N^2) \tag{87a}$$

or

$$I(0;z_p) = 2P_o/\pi\omega_{zp}^2 \tag{87b}$$

where

$$\omega_{zp}^2 = \omega^2/[1 + (\pi\omega^2/\lambda R)^2] \tag{87c}$$

Comparing Eqs. (85a) and (87a), we note that the peak axial irradiance is higher than the corresponding focal-point irradiance by $2\gamma/\pi^2 N^2$ (in units of $P_o A/\lambda^2 R^2$) or by $2P_o/\pi\omega^2$. Equations (86a) and (87c) can also be written in the form

$$z_p/R = [1 + (\pi N_g)^{-2}]^{-1} \tag{86c}$$

and

$$\omega_{zp}^2 = \omega^2/[1 + (\pi N_g)^2] \tag{87d}$$

respectively, where $N_g = \omega^2/\lambda R$ is the Gaussian Fresnel number.[12] It represents the number of Fresnel zones in the aperture plane within the Gaussian beam radius as observed from the focus, just as N represents the number of zones within the full aperture, provided that there is no obscuration. Since the axial irradiance is maximum at z_p, the beam radius is minimum at this position. This may also be seen by equating to zero the derivative of Eq. (82) with respect to z. The minimum beam radius ω_{zp} is generally referred to in the literature as the beam waist.[1]

It is evident that $z_p < R$, i.e., the peak of axial irradiance does not occur at the focal point but at a point between it and the aperture. Moreover, $\omega_{zp} < \omega$, i.e., the waist of the diffracted beam is smaller than the beam radius at the aperture. Note, however, that, as discussed in Section 4, even though the peak axial irradiance and waist are not located at the focal point $(z = R)$, the smallest beam radius and maximum central irradiance on a target at a fixed distance z are obtained when the beam is focused on it.[1]

The encircled-power distribution in an observation plane is given by

$$P(r_o;z) = 1 - \exp[-\gamma\pi^2 r_o^2/2(\Phi_o^2 + \gamma^2)] \tag{88a}$$

or

$$P(r_o;z) = P_o\, [1 - \exp[-2r_o^2/\omega_z^2)] \tag{88b}$$

where r_o is in units of $\lambda z/D$ in Eq. (88a).

The radius of curvature R_z of the Gaussian spherical wave at a distance z from the aperture is given by the inverse of the coefficient of $-i\pi r^2/\lambda$ in Eq. (1). Noting that r is in units of $\lambda z/D$ in $\beta = \pi r$, we find that

$$z/R_z = (A/\lambda z)\,\mathrm{Im}(1/\alpha) - 1$$

$$= [A\Phi_o/\lambda z(\Phi_o^2 + \gamma^2)] - 1 \tag{89a}$$

$$= \frac{1 - z/R}{(1 - z/R)^2 + (\lambda z/\pi\omega^2)^2} - 1 \tag{89b}$$

At the waist position z_p, $R_{zp} = \infty$, implying a plane wave. Similarly, at the focal plane, $R_z = -R$. A negative value of R_z indicates a diverging spherical wave.

The results for a weakly truncated collimated Gaussian beam can be obtained from those for a focused beam by letting $R \to \infty$. Thus, for example

$$I(r;z) = \{2\gamma/[1 + (4\gamma z/\pi)^2]\} \exp\{-8\gamma z^2 r^2/[1 + (4\gamma z/\pi)^2]\} \tag{90a}$$

or

$$I(r;z) = (2P_o/\pi\omega_z^2) \exp(-2r^2/\omega_z^2) \tag{90b}$$

where

$$\omega_z^2 = \omega^2[1 + (\lambda z/\pi\omega^2)^2] \tag{90c}$$

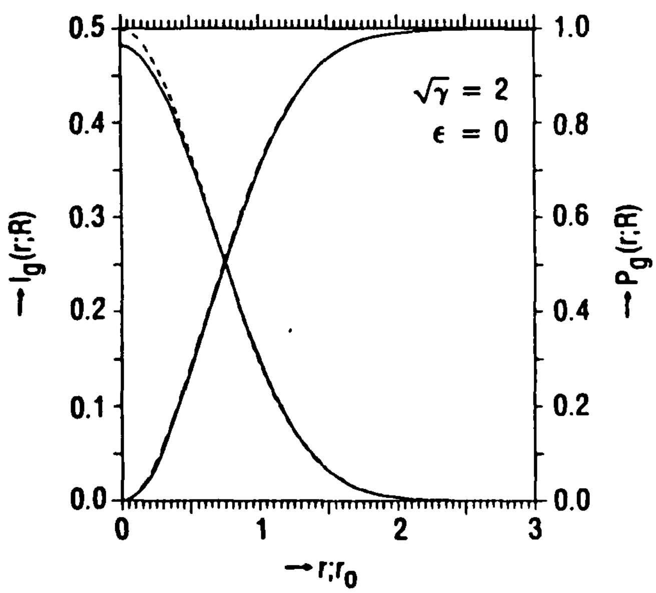
and

$$R_z = -z[1 + (\pi\omega^2/\lambda z)^2] \tag{90d}$$

In Eq. (90a), the irradiance is in units of P_o/A, r is in units of $\lambda z/D$ (z is not normalized here), and z is in units of D^2/λ. We note from Eq. (90c) that ω_z increases monotonically as z increases, i.e., the beam expands as it propagates.

In this section, we have written equations in two equivalent forms. Equations (a) are written in a normalized form so that they can be investigated parametrically. Equations (b) are written without any normalizations, and they are more suitable for evaluating results when the specific parameters involved are known. Incidentally, note that all equations in this section reduce to the corresponding quantities in the aperture plane if we let $z \to 0$.

Figure 9 shows the focal-plane irradiance and encircled-power distributions for $\sqrt{\gamma} = 2$. The solid curves have been obtained by using Eqs. (17) and (18) [or Eqs. (30 and (32) with $\Phi_o = 0$], and the dashed curves represent their corresponding approximations given by Eqs. (81a) and (88a), respectively, with $\Phi_o = 0$. We note that the approximate results agree well with the true results. The maximum difference, which occurs at the focus, is less than 4%. For larger γ, the agreement is found to be even better.



Note: The irradiance and encircled power are in units of P_0A/λ^2R^2 and P_0, respectively. The radial distance r or r_0 in the focal plane are in units of $\lambda R/D$. The focal point is at r = 0.

Figure 9. Focal-Plane Irradiance and Encircled-Power Distributions for a Gaussian Beam with $\sqrt{\gamma} = 2$

Figure 10 shows how the axial irradiance of a focused Gaussian beam varies when $\sqrt{\gamma} = 2$ and N = 1, 10, and 100. The irradiance is in units of $P_o A/\lambda^2 R^2$. Once again, the solid curves in this figure have been obtained by using Eq. (24), and the dashed curves represent their corresponding approximations given by Eq. (83a). It is evident that Eq. (83a) represents the true axial irradiance quite well. The only significant difference occurs when N = 1, in that the true results show secondary maxima and minima but the approximate result shows only the principal maximum. For larger values of γ, e.g., $\sqrt{\gamma} = 2.5$, the secondary maxima and minima disappear, and the true and approximate results overlap each other at the scale of Figure 10. Hence, we conclude that the truncation of an aberration-free Gaussian beam by an aperture has a negligible effect on the irradiance distribution as the beam propagates when $\sqrt{\gamma} \geq 2$.

7.2 ABERRATION BALANCING

When a Gaussian circular beam is weakly truncated, i.e., when γ is large, the quantity p_s in Table 2 reduces to

$$\begin{aligned} p_s &= \langle \rho^s \rangle \\ &= (s/2\gamma)\, p_{s-2} \\ &= (s/2)!\, \gamma^{-s/2} \end{aligned} \tag{91}$$

As a result, we obtain simple expressions for the radial polynomials representing balanced primary aberrations. They are listed in Table 2. If we normalize ρ by ω (instead of by a as in Table 2), then γ disappears from these expressions. As in Section 6, the standard deviation of an aberration can be obtained by comparing its form with the corresponding orthonormal aberration of Eq. (43).

The standard deviation of primary aberrations and of the corresponding balanced aberrations for a weakly truncated Gaussian beam is given in Table 4. In this table, $\rho' = \sqrt{\gamma}\,\rho$ is a radial variable in the aperture plane normalized by the beam radius ω, and the aberration coefficients A_i' represent the peak

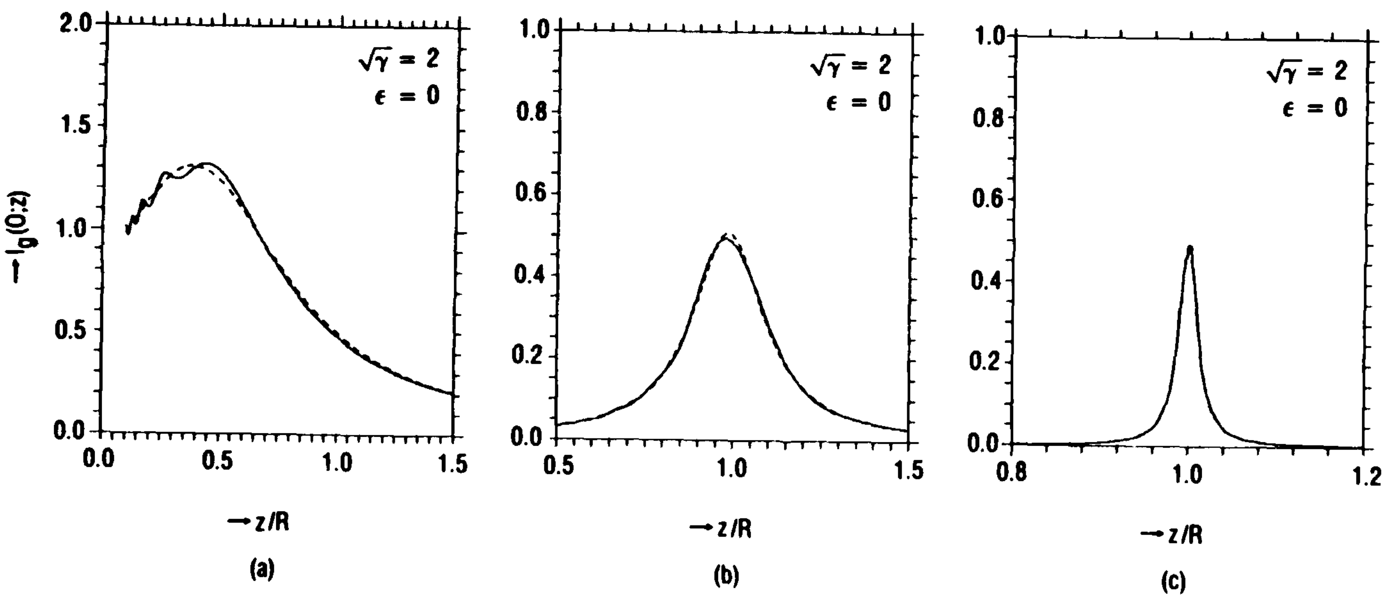


Figure 10. Axial Irradiance of a Gaussian Beam with $\sqrt{\gamma} = 2$ Focused at a Distance R with a Fresnel Number N = 1, 10, and 100

value of a classical aberration at $\rho' = 1$. The reason for defining the aberration coefficients in this manner should be obvious. Since the power in a very weakly truncated Gaussian beam is concentrated in a small region near the center of the aperture, the effect of the aberration in its outer region is negligible. Accordingly, aberration tolerances in terms of the peak value of the aberration at the edge of the aperture are not very meaningful.

We note from Table 2 (or Table 4) that the point with respect to which the variance of an aberration is minimized is given by

$$\Phi_o = -(4/\gamma)A_s \tag{92a}$$

$$= -4\gamma \, A'_s$$

$$A_t = -(2/\gamma)A_c \tag{92b}$$

$$= -2\sqrt{\gamma} \, A'_c$$

and

$$\Phi_o = -(1/2)A_a$$

$$= -(\gamma/2)A'_a \tag{92c}$$

in the case of spherical aberration, coma, and astigmatism, respectively. From Table 4, we note that the balancing of a primary aberration reduces its standard deviation by a factor of $\sqrt{5}$, $\sqrt{3}$, and $\sqrt{2}$ in the case of spherical aberration, coma, and astigmatism, respectively. These reduction factors are listed in Table 5 for uniform ($\gamma = 0$) beam, and e^{-2} truncated ($\gamma = 1$) and weakly truncated (large γ) Gaussian beams. It is evident that as γ increases, the reduction factors decrease for each of the three primary aberrations. The amount of balancing aberration decreases as γ increases in the case of spherical aberration and coma, but it does not change in the case of astigmatism. For example, in the case of spherical aberration, the amount of balancing defocus for a weakly truncated Gaussian beam is $(4/\gamma)$ times the corresponding amount for a uniform beam. Similarly, in the case of coma, the balancing tilt for a weakly truncated Gaussian beam is $(3/\gamma)$ times the corresponding amount for a uniform beam.

Table 4. Standard Deviation of Primary Aberrations in Weakly Truncated ($\sqrt{\gamma} \geq 3$) Gaussian Circular Beams

Aberration, $\Phi(\rho, \theta)$	σ_Φ
Spherical	
$A'_s \rho'^4$	$2\sqrt{5}\, A'_s$
Balanced Spherical	
$A'_s(\rho'^4 - 4\rho'^2)$	$2\, A'_s$
Coma	
$A'_c\, \rho'^3 \cos\theta$	$\sqrt{3}\, A'_c$
Balanced Coma	
$A'_c(\rho'^3 - 2\rho')\cos\theta$	A'_c
Astigmatism	
$A'_a \rho'^2 \cos^2\theta$	$(1/\sqrt{2})\, A'_a$
Balanced Astigmatism	
$A'_a \rho'^2 \cos^2\theta - (1/2) A'_a \rho'^2$ $= (1/2)\, A'_a\, \rho'^2 \cos 2\theta$	$(1/2)\, A'_a$

$\rho' = \sqrt{\gamma}\rho$, $A'_s = A_s/\gamma^2$, $A'_c = A_c/\gamma^{3/2}$, $A'_a = A_a/\gamma$, $A'_d = A_d/\gamma$, $A'_t = A_t/\sqrt{\gamma}$.

Table 5. Standard Deviation Reduction Factor for Primary Aberrations*

Aberration	Reduction Factor		
	Uniform ($\gamma = 0$)	Gaussian ($\gamma = 1$)	Weakly Truncated Gaussian ($\sqrt{\gamma} \geq 3$)
Spherical	4	3.74	2.24
Coma	3	2.64	1.73
Astigmatism	1.22	1.66	1.41

*It represents the factor by which the standard deviation of a classical aberration across a circular aperture is reduced when it is optimally balanced with other aberrations.

Table 6 gives the reduction factors that relate the peak value A_i of a primary aberration at the edge of a circular aperture and the standard deviation of its corresponding balanced aberration for various values of γ. In the case of balanced aberrations, these numbers are given by $\sqrt{5}\ a_4^0$, $2\sqrt{2}\ a_3^1$, and $2\sqrt{6}\ a_2^2$ for spherical aberration, coma, and astigmatism, respectively. For example, for spherical aberration $A_s\rho^4$, the standard deviation of the corresponding balanced spherical aberration when $\sqrt{\gamma} = 2$ is equal to $A_s/18.29$.

Comparing the standard deviation results given in Table 6 with those for a weakly truncated Gaussian beam in Table 4, we find that they agree with each other with negligible difference for $\sqrt{\gamma} > 3$. This provides a convenient definition for a weakly truncated Gaussian beam, namely, that $a > 3\omega$. Some authors[30,31] have assumed that $\sqrt{\gamma} \geq 2$ provides a sufficient condition for the validity of the aberration analysis of a weakly truncated Gaussian beam given here. When $\sqrt{\gamma} = 2$, the standard deviation of balanced spherical aberration according to the weakly truncated beam assumption is given by $A_s/8$, while the true value, as stated above, is given by $A_s/18.29$, which is significantly different. When $\sqrt{\gamma} = 3$, the corresponding standard deviations are given by $A_s/40.50$ and $A_s/43.52$, which are nearly equal to each other. The difference between the true and approximate results is even less for $\sqrt{\gamma} > 3$.

When $\sqrt{\gamma} = 2$, even though the true focal-plane distribution obtained from Eq. (17) or Eq. (30) agrees quite well with that obtained from Eq. (84a), the true and approximate standard deviations of primary aberrations are significantly different as pointed out above. The reason for the discrepancy in the case of an aberrated beam is simple. Even though the irradiance in the region of the aperture $\omega/a \leq \rho \leq 1$ is quite small compared to that at or near its center, the amplitude in this region is not as small. Moreover, the aberration in this region can be quite large and thus have a significant effect on the standard deviation. In the case of spherical aberration, it increases

[30]Yoshida, A., "Spherical Aberration in Beam Optical Systems," Appl. Opt., 21, 1982, pp. 1812-1816.

[31]Herloski, R., "Strehl Ratio for Untruncated Aberrated Gaussian Beams," J. Opt. Soc. Am., A2, 1985, pp. 1027-1030.

Table 6. Standard Deviation Factor for Primary Aberrations for a Gaussian Circular Beam with Various Values of γ

$\sqrt{\gamma}$	Balanced Spherical	Balanced Coma	Balanced Astigmatism
0	13.42	8.49	4.90
0.5	13.69	8.53	5.06
1.0	13.71	8.80	5.61
1.5	14.90	9.74	6.81
2.0	18.29	12.21	9.08
2.5	26.33	17.62	12.82
3.0	43.52	27.57	18.06
3.5	75.78	42.96	24.51
4.0	128.09	64.01	32.00

Note: The numbers represent the factor by which the peak aberration coefficient A_i must be divided by in order to obtain the standard deviation.

as ρ^4. In the case of coma and astigmatism, it increases as ρ^3 and ρ^2, respectively. Hence, we require a larger value of γ, namely, $\sqrt{\gamma} \geq 3$, for the aberrated-beam analysis of this section to be valid. This is also true of defocus which varies as ρ^2.

8. CONCLUSIONS

We have compared the effects of diffraction, obscuration, and aberrations on the propagation of uniform and Gaussian beams.[32] This comparison is based on fixed total power in the beam regardless of the obscuration or the amplitude across it at the aperture plane. The following general conclusions can be drawn from the work reported here.

The mirrors in a high-power Gaussian beam train are illuminated unevenly and must withstand higher flux densities than those in a corresponding uniform beam train. The uneven illumination will cause thermal distortions of the mirrors thereby introducing aberrations in the beam.

The focal-point irradiance for a focused Gaussian beam is smaller than the corresponding value for a uniform beam of the same total power. Also, the encircled power for small circles is higher for a uniform circular beam, but for large circles, it is higher for a Gaussian circular beam.

The effect of the Gaussian apodization of the aperture is to increase the size of the central disc and to decrease the power in the rings of the diffraction pattern. The effect of a central obscuration is opposite to that of the Gaussian apodization. It reduces the size of the central disc and increases the power in the rings of the diffraction pattern. Accordingly, as the obscuration increases, the difference between the diffraction effects of uniform and Gaussian beams decreases.

The minima of axial irradiance for a uniform beam have a value of zero, while those for a Gaussian beam have nonzero values. While the principal maximum of axial irradiance for a Gaussian beam has a smaller value than the corresponding value for a uniform beam, the secondary maxima for a Gaussian beam have higher values. Even though the principal maximum does not necessarily

[32] Mahajan, V. N., "Comparison of Uniform and Gaussian Beam Diffraction," SPIE Proc., Diffraction Phenomena in Optical Engineering Applications, 560, 1985.

occur at the focus, maximum central irradiance and encircled energy are obtained on a target at a given distance from the aperture when the beam is focused on it, unlike the conclusions of some authors.[33]

For $a < \omega$, the Gaussian beams are somewhat less sensitive to aberrations than uniform beams. Accordingly, aberration tolerance is somewhat higher for the Gaussian beams. However, this tolerance increases rapidly as a becomes larger and larger compared to ω. This is understandable since, for a much greater than ω, the power in the aperture is concentrated in a small region near the center of the aperture; therefore, the aberration in its outer region has little effect on the irradiance distribution. For $a \geq 2\omega$, the truncation of an aberration-free Gaussian beam by the aperture has a negligible effect on its propagation. Accordingly, it remains a Gaussian beam as it propagates. However, when the beam is aberrated, $a \geq 3\omega$ is required in order to neglect the effect of its truncation by the aperture. When $a \geq 3\omega$, it is more appropriate to define aberration coefficients as the peak aberrations at the beam radius ω rather than at the aperture edge a, since the power in the beam is concentrated in a small region near the center of the aperture, and the effect of an aberration in its outer region is negligible. With the aberration coefficients defined in this manner, the beam becomes most sensitive to spherical aberration and least sensitive to astigmatism, rather than being most sensitive to coma and least sensitive to spherical aberration.

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[33] Tanaka, K., N. Saga, and K. Hauchi, "Focusing of a Gaussian Beam Through a Finite Aperture Lens," Appl. Opt., 24, 1985, pp. 1098-1101.

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